DEPARTMENT OF MATHEMATICS

# Mathematics for Computer Science

## (COURSE CODE 23MATS31)

## MODULE-I

## PROBABILITY DISTRIBUTION

**Discrete and continuous Random Variables, Probability density function and distributions. Binomial, Poisson, and Normal distributions.**

**Random Experiment:** An activity that yield some results called the random experiment.The random variable means a real number i.e. X associated with the outcomes of a random experiment.

**Random variable:**

If a real variable X is associated with every outcome of a random experiment, then X is called a random variable or a stochastic variable or simply a variate. i.e., A random variable X is a function from the set S (sample space) to the set of real numbers R. i.e., $X=f(s) \quad \forall\ s \in S$.

The random variables are two types,they are

i)Discrete Random Variables(DRV)

ii)Continuous Random Variables(CRV)

Discrete Random Variables: A Disecrete random variable is a variable which can only take a countable number of values.

**Example:**

Suppose a coin is tossed twice, then the sample space is S= {HH, HT, TH, TT}. Let X denote the number of heads turned up i.e., X= {2,1,1,0}, then X is the random variable. Therefore, the range of X= {0,1,2}.

Continuous Random Variables: A Continuous random variable is a random variable where the data can take infinitely many values.For example, a random  variable measuring the time taken for something to be done is continuous since there are an infinite number of possible times that can be taken.

Example:Temperature of the climate,Age of a person,etc.

## Discrete Probability Distribution:

Let X be the discrete random variable. If the probability that X takes the values $x_i$ is $p_i$, then $P(X = x_i) = p_i$ or $P(x_i)$ for $i = 1, 2, \dots \dots$, where (i) $P(x_i) \geq 0, \quad \forall\, i$. (ii) $\sum P(x_i) = 1$.
The set of values $x_i$ with their probabilities $p_i$ constitute a *discrete probability distribution* of the discrete variate X. The functions P(X) is called the *probability density function (p.d.f)* or theprobability *mass function (p.m.f).*

## Distribution function:

The distribution function $F(x)$ of the discrete variate X is defined by
$F(x) = P(X \leq x) = \sum_{i=1}^{x} P(x_i)$, where $x$ is any integer. $F(x)$ is also known as *cumulative distribution function(c. d. f).*

## Expectation and variance:

The mean value $\mu$ of the probability distribution of variate X is called the expectation of it and it is denoted by E[X].

The mean and variance of the discrete probability distribution is defined as follows.

Mean $\mu = E[X] = \sum_i x_i\, P(x_i)$ and variance (V) of the distribution is

$\sigma^2 = \sum_i (x_i - \mu)^2\, P(x_i) = \sum_i x_i^2\, P(x_i) - \mu^2$. i.e., $Var(X) = E[X^2] - \{E[X]\}^2$,

where $E[X^2] = \sum x_i^2\, P(x_i)$. $\therefore$ Standard deviation of the distribution is $\sigma = \sqrt{Var(X)}$.

## Problems:

1. **A coin is tossed twice. A random variable X represents the number of heads turning up. Find the discrete probability distribution for X. Also, find its mean and variance.**

**Solution:**
The sample space $S = \{HH, HT, TH, TT\}$. Let X = number of heads turned up.

$\therefore P(X = 0) = P\,(TT) = P(\text{no head turned up}) = \frac{1}{4}$.

$P(X = 1) = P\,(HT) + P(TH) = P(\text{one head turned up}) = \frac{2}{4} = \frac{1}{2}$.

$P(X = 2) = P\,(HH) = P(\text{two heads turned up}) = \frac{1}{4}$.

$\therefore$ The probability distribution is

| $X = x_i$ | 0 | 1 | 2 |
|---|---|---|---|
| | | | |

| $P(x_i)$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ |
|---|---|---|---|

We observe that (i) $P(x_i) \geq 0$ . (ii) $\sum P(x_i) = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1$.

∴ It is a discrete probability distribution.

∴ Mean $\mu = \sum_i x_i \, P(x_i) = 0\left(\frac{1}{4}\right) + 1\left(\frac{1}{2}\right) + 2\left(\frac{1}{4}\right) = 1$.

Variance $V = \sigma^2 = \sum_i x_i^2 \; P(x_i) - \mu^2 = 0^2\left(\frac{1}{4}\right) + 1^2\left(\frac{1}{2}\right) + 2^2\left(\frac{1}{4}\right) - 1 = \frac{1}{2}$.

**2. A die is tossed thrice. A success is getting 1 or 6 in a toss. Find the mean and variance of the number of success.**

**Solution:**

Probability of success $= \frac{2}{6} = \frac{1}{3}$, Probability of failure $= 1 - \frac{1}{3} = \frac{2}{3}$.

∴ $P(X = 0) = P(\text{no success in all toss}) = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$ .

$$P(X = 1) = P(\text{one success and two failures}) = \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = 3\left(\frac{4}{27}\right)$$

$$= \frac{4}{9}$$

$$P(\text{two successes and one failure}) = \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} = 3\left(\frac{2}{27}\right)$$

∴ $P(X = 2) = \frac{2}{9}$

$P(X = 3) = P(\text{success in all 3 tosses}) = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$ .

∴ The probability distribution is

| $X = x_i$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(x_i)$ | $\frac{8}{27}$ | $\frac{4}{9}$ | $\frac{2}{9}$ | $\frac{1}{27}$ |

∴ Mean $\mu = \sum_i x_i \, P(x_i) = 0\left(\frac{8}{27}\right) + 1\left(\frac{4}{9}\right) + 2\left(\frac{2}{9}\right) + 3\left(\frac{1}{27}\right) = 1$

Variance $V = \sigma^2 = \sum_i x_i^2 \; P(x_i) - \mu^2 = 0^2\left(\frac{8}{27}\right) + 1^2\left(\frac{4}{9}\right) + 2^2\left(\frac{2}{9}\right) + 3^2\left(\frac{1}{27}\right) - 1 = \frac{2}{3}$.

**3. The probability density function of a variate X is**

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| P(X) | $k$ | $3k$ | $5k$ | $7k$ | $9k$ | $11k$ | $13k$ |

**(i) Find $P(X < 4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$**

**(ii) What will be the minimum value of $k$ so that $P(X \leq 2) \geq 0.3$**

**Solution:**

**(i)** Since P(X) is the p. d. f, we have $P(x) > 0$ and $\sum P(x_i) = 1$.

$\therefore\ k > 0$ and $k + 3k + 5k + 7k + 9k + 11k + 13k = 1$. $\therefore\ k = \frac{1}{49}$.

$P(X < 4) = P(0) + P(1) + P(2) + P(3) = k + 3k + 5k + 7k = 16k = \frac{16}{49}$ .

$$P(X \geq 5) = P(5) + P(6) = 11k + 13k = 24k = \frac{24}{49}$$

$$P(3 < X \leq 6) = P(4) + P(5) + P(6) = 9k + 11k + 13k = \frac{33}{49}$$

**(ii)** $P(X \leq 2) \geq 0.3 \Rightarrow P(0) + P(1) + P(2) \geq 0.3$. $\therefore k + 3k + 5k \geq 0.3$. $\therefore 9k \geq 0.3$

$\therefore\ k \geq \frac{0.3}{9}$. $\therefore\ k \geq \frac{1}{30}$. $\therefore$ The minimum value of $k = \frac{1}{30}$.

**4. A random variable X has the following probability function:**

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| P(X) | 0 | $k$ | $2k$ | $2k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2 + k$ |

**(i) Find $k$** **(ii) Evaluate $P(X < 6)$ and $P(X \geq 6)$,** **(iii) $P(3 < X \leq 6)$**

**Solution:**

**(i)** Since P(X) is the p. d. f, we have $P(x) > 0$ and $\sum P(x_i) = 1$.

$\therefore\ k > 0$ and $0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$.

$$i.e., 10k^2 + 9k - 1 = 0\ or\ (10k - 1)(k + 1) = 0$$

$\therefore k = {}^{1}/_{10}\ and\ k = -1$ if $k = -1$ the first condition fails $\therefore\ k = \frac{1}{10}$

(ii) $P(X < 6) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) = \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100} = \frac{81}{100}$

$$P(X \geq 6) = P(6) + P(7) = \frac{2}{100} + \frac{17}{100} = \frac{19}{100}$$

(iii) $P(0 < X < 5) = P(4) + P(5) + P(6) = \frac{3}{10} + \frac{1}{100} + \frac{2}{100} = \frac{33}{100}$.

**HOME WORK:**

**2. A random variable X has the following probability function:**

| **X** | **-2** | **-1** | **0** | **1** | **2** | **3** |
|---|---|---|---|---|---|---|
| **P(X)** | **0.1** | $k$ | **0.2** | $2k$ | **0.3** | $k$ |

**(i) Find $k$  (ii) calculate mean, variance, and standard deviation.**

Ans: (i) $k = 0.1$  (ii) $\mu = 0.8$, $\sigma^2 = 2.16$, and $\sigma = 1.47$.

**Binomial Distribution:**

If we perform a series of independent trials such that for each trial, $p$ is the probability of a success and $q$ is the probability of a failure, then the probability of $x$ successes in a series of $n$ trials is given by, $P(x) = nC_x p^x q^{n-x}$, where $x = 0, 1, 2, 3, \ldots\ldots n$ and $p + q = 1$.

The probability of number of success so obtained is called the binomial distribution which is given by

| $x$: | 0 | 1 | 2 | ...... | $n$ |
|---|---|---|---|---|---|
| P($x$) : | $q^n$ | $nC_1 p^1 q^{n-1}$ | $nC_2 p^2 q^{n-2}$ | ....... | $p^n$ |

We note that (i) $P(x) \geq 0$ and (ii) $\sum P(x) = 1$.

**Binomial Theorem:** If $n$ is a positive integer, then
$(a+b)^n = a^n + nC_1 a^{n-1} b + nC_2 a^{n-2} b^2 + nC_3 a^{n-3} b^3 + \ldots . + b^n$.

**Constants of Binomial Distribution:**

(i) Mean = $\mu = E[x] = \sum x\, P(x) = \sum_{x=0}^{n} x\,.\,nC_x p^x q^{n-x}$

$$\therefore \quad \mu = 0.nC_0 p^0 q^n + 1.nC_1 p^1 q^{n-1} + 2.nC_2 p^2 q^{n-2} + 3.nC_3 p^3 q^{n-3} + \cdots + n.nC_n p^n q^0$$

$$= npq^{n-1} + 2.\frac{n(n-1)}{2.1} p^2 q^{n-2} + 3.\frac{n(n-1)(n-2)}{3.2.1} p^3 q^{n-3} + \cdots + n.p^n$$

$$= np\left[q^{n-1} + (n-1)pq^{n-2} + \frac{(n-1)(n-2)}{2.1} p^2 q^{n-3} + \cdots + p^{n-1}\right]$$

$$= np\left[(n-1)C_0 q^{n-1} + (n-1)C_1 pq^{n-2} + (n-1)C_2 p^2 q^{n-3} + \cdots + (n-1)C_{(n-1)} p^{n-1}\right]$$

$$= np(q+p)^{n-1} = np \ \because \ p+q=1$$

$\therefore \mu = np.$

(ii) Variance = V = $\sigma^2 = \sum x^2 P(x) - \mu^2$-------- (1)

Now, $\sum x^2 P(x) = \sum[x(x-1)+x]P(x) = \sum x(x-1)P(x) + \sum xP(x).$

$= \sum_{x=0}^{n} x(x-1)\, .nC_x p^x q^{n-x} + \mu.$

$$= 0 + 0 + 2.1.nC_2 p^2 q^{n-2} + 3.2.nC_3 p^3 q^{n-3} + \cdots + n(n-1)p^n + \mu$$

$$= 2.1.\frac{n(n-1)}{2.1} p^2 q^{n-2} + 3.2.\frac{n(n-1)(n-2)}{3.2.1} p^3 q^{n-3} + \cdots + n(n-1)p^n + \mu$$

$$= n(n-1)p^2[q^{n-2} + (n-2)C_1 pq^{n-3} + \cdots + p^{n-2}] + \mu$$

$= n(n-1)p^2(q+p)^{n-2} + \mu = n(n-1)p^2 + \mu$ put in (1)

$$\therefore \ \mathrm{V} = n(n-1)p^2 + \mu - \mu^2 = n^2p^2 - np^2 + np - n^2p^2 = np - np^2$$

$$= np(1-p) = npq \because p+q=1$$

$$\therefore \ \mathrm{V} = npq$$

$\therefore$ Standard deviation $= \sigma = \sqrt{V} = \sqrt{npq}$

**Application of Binomial Distribution:**

This distribution is applied to problems concerning

(i) Number of defectives in a sample from production line.
(ii) Estimation of reliability of systems.
(iii) Number of rounds fixed from a gun hitting a target.
(iv) Radar detection.

**Problems:**

**1. The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that (i) exactly two will be defective (ii) at least two will be defective (iii) none will be defective.**

**Solution:**

Here $p = \frac{1}{10} = 0.1$, $q = 1 - p = 1 - 0.1 = 0.9$ and $n = 12$.

The binomial distribution function is

$$P(x) = nC_x p^x q^{n-x} = 12\ C_x (0.1)^x (0.9)^{12-x},\ \ x = 0, 1, 2, \dots, 12$$

(i) P (exactly two pens will be defective) = $P(x = 2) = 12\ C_2 (0.1)^2 (0.9)^{10} = 0.2301$

(ii) P (at least two pens will be defective) = $P(x \geq 2) = 1 - P\ (x < 2)$

$$= 1 - [P(0) + P(1)] = 1 - 12\ C_0 (0.1)^0 (0.9)^{12} - 12\ C_1 (0.1)^1 (0.9)^{11} = 0.3409$$

(iii) P (none will be defective) = $P(x = 0) = 12\ C_0 (0.1)^0 (0.9)^{12} = 0.2824$.

**2. The probability that a man aged 60 will live up to 70 is 0.65. What is the probability that out of 10 man, now aged 60 (i) exactly 9 will live to be 70, (ii) at most 9 will live to be 70, (iii) at least 7 will live to be 70?**

**Solution:**

Here $p = 0.65,\ \ q = 1 - p = 0.35, n = 10$. The binomial distribution function is

$$P(x) = nC_x p^x q^{n-x} = 10\ C_x (0.65)^x (0.35)^{10-x},\ \ x = 0, 1, 2, \dots, 10$$

(i) P (exactly 9 will live to be 70) = $P(x = 9) = 10\ C_9 (0.65)^9 (0.35)^1 = 0.0724$

(ii) P (at most 9 will live to be 70) = $P(x \leq 9) = 1 - P\ (x > 9)$

$$= 1 - P(10) = 1 - 10\ C_{10} (0.65)^{10} (0.35)^0 = 0.9865$$

(iii) P (at least 7 will live to be 70) = $P(x \geq 7) = P(7) + P(8) + P(9) + P(10) = 0.5138$.

**3. In sampling a large number of parts manufactured by a machine, the mean number of defective in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain at least 3 defective parts.**

**Solution:**

Here$n = 20,\ \ \mu = 2.\ \therefore\ \mu = np\ \Rightarrow p = \frac{\mu}{n} = \frac{2}{20} = 0.1, q = 1 - p = 0.9$.

$\therefore$ The binomial distribution function is

$$P(x) = nC_x p^x q^{n-x} = 20\ C_x (0.1)^x (0.9)^{20-x},\ \ x = 0, 1, 2, \dots, 20$$

P (at least 3 defective parts in the sample) = $P(x \geq 3) = 1 - P(x < 3)$

$$= 1 - [P(0) + P(1) + P(2)] = 0.323$$

$\therefore$ The number of samples having at least 3 defective parts out of 1000 samples $= 1000 \times 0.323$

$= 323$.

**4. In 256 sets of 12 tosses of a coin, in how many cases one can expect 8 heads and 4 tails.**

**Solution:**

Let$p = P(head) = \frac{1}{2}$, $q = P(tail) = \frac{1}{2}$, $n = 12$. The binomial distribution function is

$P(x) = nC_x p^x q^{n-x} = 12\,C_x (0.1)^x (0.9)^{12-x}$, $x = 0, 1, 2, \ldots$

$\therefore$ Take $x = 8\ (head)$ $\therefore$ $n - x = 12 - 8 = 4\ (tail)$

$\therefore$ By binomial distribution, $P(x = 8) = 12C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^4 = \frac{495}{4096}$

$\therefore$ The expected number of such cases in 256 sets $= 256 \times \frac{495}{4096} = 30.9 \approx 31.$

**5. Out of 800 families with 5 children each, how many would you expect to have (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys? Assume equal probabilities for boys and girls.**

**Solution:**

Let $p = P(boy\ child) = \frac{1}{2}$, $q = P(girl\ child) = \frac{1}{2}$, $n = 5$,

Binomial distribution function is

$$P(x) = nC_x p^x q^{n-x} = 5C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x} = 5C_x \left(\frac{1}{2}\right)^5 = \frac{1}{32} 5C_x$$

(i)$P(3) = \frac{1}{32} 5C_3 = 0.3125$

$\therefore$ Number of families have 3 boys $= 800 \times 0.3125 = 250$

(ii)$P(0) = \frac{1}{32} 5C_0 = 0.03125$

$\therefore$ Expected number of families have 5 girls $= 800 \times 0.03125 = 25$

(iii)$P(2) + P(3) = \frac{1}{32} 5C_2 + \frac{1}{32} 5C_3 = 0.3125 + 0.3125 = 0.625$

$\therefore$ Expected number of families have either 2 or 3 boys $= 800 \times 0.625 = 500.$

**HOME WORK:**

**1. The number of telephone lines busy at an instant of time is a binomial variate with probability 0.2. If at an instant 10 lines are chosen at random, what is the probability that (i) 5 lines are busy (ii) at moat two lines are busy (iii) all lines are busy?**

**Ans:** (i) 0.0264 (ii) 0.6778 (iii) $1.024 \times 10^{-7}$.

**2. If 10 percent of the rivets produced by a machine are defective. Find the probability that out of 5 rivets chosen at random (i) none will be defective (ii) one will be defective (iii) at least two will be defective.**

**Ans:** (i)0.5905 (ii) 0.328 (iii) 0.0815

**3. The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$. If six bombs are dropped, find the probability that (i) exactly two will strike the target (ii) at least two will strike the target.**

**Ans:**(i) 0.246 (ii) 0.345.

**4. in quiz program of answering YES or NO, what is the probability of guessing atleast 6 answers correctly out of 10 questions? Also find the probability of the same if there are four options for a correct answer.**

**Ans :** 0.37721 and 0.01972.

## Poisson Distribution:

The Poisson distribution is related to the probabilities of events which are extremely rare, but which have a large number of independent opportunities of occurrence.

The Poisson distribution function is given by $P(x) = \frac{\mu^x e^{-\mu}}{x!}$ , where $\mu = np$, $x = 0, 1, 2, \ldots$ and the corresponding distribution is given by

| $x$: | 0 | 1 | 2 | …… |
|---|---|---|---|---|
| P$(x)$ : | $e^{-\mu}$ | $\frac{e^{-\mu}}{1!}$ | $\frac{\mu^2 e^{-\mu}}{2!}$ | ……. |

We note that (i) $P(x) \geq 0, \quad \forall\, x$ (ii) $\sum P(x) = 1$.

NOTE: Poissondistribution is a limiting case of a binomial distribution.

## Constants of Poisson distribution:

(i) Mean = E[x] = $\sum x\, P(x) = \sum_{x=0}^{\infty} x. \ \frac{\mu^x e^{-\mu}}{x!}$ , $x = 0,1,2, \ldots$

$$= 0 + \frac{\mu e^{-\mu}}{1!} + 2\,\frac{\mu^2 e^{-\mu}}{2!} + 3\,\frac{\mu^3 e^{-\mu}}{3!} + \cdots$$

$$= \mu e^{-\mu} + \frac{\mu^2 e^{-\mu}}{1!} + \frac{\mu^3 e^{-\mu}}{2!} + \cdots$$

$= \mu e^{-\mu}\left[1+\frac{\mu}{1!}+\frac{\mu^2}{2!}+\cdots\right] = \mu e^{-\mu}.e^{\mu} = \mu \qquad [\because e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\ldots\ldots]$

$\therefore$ Mean $= \mu = np \qquad [\because \mu = np]$

(ii) Variance = V = $\sum x^2P(x) - \mu^2$ ------- (1)

Now, $\sum x^2P(x) = \sum_{x=0}^{\infty}[x(x-1)+x]P(x) = \sum_{x=0}^{\infty} x(x-1)P(x) + \sum_{x=0}^{\infty} xP(x)$

$$= \sum_{x=0}^{\infty} x(x-1)\frac{\mu^x e^{-\mu}}{x!} + \mu$$

$$= \left[0+0+2.1.\frac{\mu^2e^{-\mu}}{2.1} + 3.2.\frac{\mu^3e^{-\mu}}{3.2.1} + 4.3\,\frac{\mu^4e^{-\mu}}{4.3.2.1} + \cdots\right] + \mu$$

$$= \mu^2e^{-\mu}\left[1+\frac{\mu}{1!}+\frac{\mu^2}{2!}+\cdots\right] + \mu = \mu^2e^{-\mu}e^{\mu} + \mu$$

$= \mu^2 + \mu.$ Put in (1)

$$\therefore \text{ V } = \mu^2 + \mu - \mu^2 = \mu. \therefore \text{ V } = np \therefore \text{ S.D} = \sigma = \sqrt{np}$$

## Application of Poisson distribution:

This distribution is applied to the problems concerning,

(i) Arrival pattern of "defective vehicles in a workshop", "patients in a hospital" or "telephone calls".

(ii) Demand pattern for certain spare parts.

(iii) Number of fragments from a shell hitting a target.

(iv) Spatial distribution of bomb hits.

## Problems:

**1. If a random variable has a Poisson distribution such that $P(1) = P(2)$, find (i) mean of the distribution (ii) $P(4)$**

**Solution:**

Poisson distribution is given by the function, $P(x) = \frac{\mu^x e^{-\mu}}{x!}$, $x = 0, 1, 2, \ldots\ldots$

(i) Given $P(1) = P(2)$ $\quad\therefore \frac{\mu e^{-\mu}}{1!} = \frac{\mu^2 e^{-\mu}}{2!}$ $\quad\therefore \frac{1}{1} = \frac{\mu}{2}$ $\quad\therefore \mu = 2.$

(ii) $P(4) = \frac{2^4e^{-2}}{4!} = 0.0902.$

**2. If the probability of a bad reaction from a certain injection is 0.001. Determine the**

**chance that out of 2000 individuals (i) more than 2 will get a bad reaction (ii) exactly 3 will get a bad reaction.**

**Solution:**

Here $p = 0.001, \quad n = 2000 \therefore \quad \mu = np = 2000 \times 0.001 = 2.$

Poisson distribution is given by $P(x) = \frac{\mu^x e^{-\mu}}{x!} = \frac{2^x e^{-2}}{x!}, \quad x = 0, 1, 2, \ldots$

(i) P (more than 2 will get a bad reaction) = $P(x > 2) = 1 - P(x \le 2)$

$$= 1 - [P(0) + P(1) + P(2)] = 1 - \frac{2^0 e^{-2}}{0!} - \frac{2^1 e^{-2}}{1!} - \frac{2^2 e^{-2}}{2!} = 0.3233$$

(ii) P (exactly 3 will get a bad reaction) = $P(x = 3) = \frac{2^3 e^{-2}}{3!} = 0.1804.$

**3. In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) one defective (iii) two defective blades respectively in a consignment of 10,000 packets.**

**Solution:**

Given $p = 0.002, \quad n = 10. \therefore \quad \mu = np = 10 \times 0.002 = 0.02$

Poisson distribution is given by $P(x) = \frac{\mu^x e^{-\mu}}{x!} = \frac{(0.02)^x e^{-0.02}}{x!}, \quad x = 0, 1, 2, \ldots$

(i) $P(0) = e^{-0.02} = 0.9802.$

$\therefore$ The number of packets containing no defective blade = $10{,}000 \times 0.9802 = 9802$

(ii) $P(1) = \frac{(0.02) e^{-0.02}}{1!} = 0.0196.$

$\therefore$ The number of packets containing one defective blade = $10{,}000 \times 0.0196 = 196$

(iii) $P(2) = \frac{(0.02)^2 e^{-0.02}}{2!} = 0.000196.$

$\therefore$ The number of packets containing two defective blades = $10{,}000 \times 0.000196 = 1.96 \approx 2.$

**4. A car hire firm has 2 cars, which it hires out day by day. The demand for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the probability that on a certain randomly chosen day (i) neither car is used and (ii) some demand is refused.**

**Solution:**

Given $\mu = 1.5. \therefore P(x) = \frac{\mu^x e^{-\mu}}{x!} = \frac{(1.5)^x e^{-1.5}}{x!}, \quad x = 0, 1, 2, \ldots$

(i) P (neither car is used) = $P(x = 0) = e^{-1.5} = 0.2231.$

(ii) P (some demand is refused) = $P(x > 2) = 1 - P(x \leq 2) = 1 - [P(0) + P(1) + P(2)]$

$= 1 - \left[\frac{(1.5)^0 e^{-1.5}}{0!} + \frac{(1.5)^1 e^{-1.5}}{1!} + \frac{(1.5)^2 e^{-1.5}}{2!}\right] = 0.1913.$

**5. Given that 2% of the fuses manufactured by a firm are defective, find by using Poisson distribution, the probability that a box containing 200 fuses has (i) no defective fuse (ii) at least one defective fuse (iii) exactly 3 defective fuses (iv) 3 or more defective fuses (v) between 2 or 4 defective fuses.**

**Solution:**

Given$p = \frac{2}{100} = 0.02, \quad n = 200 \therefore \quad \mu = np = 200 \times 0.02 = 4$

Poisson distribution is given by $\quad P(x) = \frac{\mu^x e^{-\mu}}{x!} = \frac{(4)^x e^{-4}}{x!}, \quad x = 0, 1, 2, \ldots\ldots\ldots$

(i) $P(0) = \frac{(4)^0 e^{-4}}{0!} = 0.0183.$

(ii) $P(x \geq 1) = 1 - P(x < 1) = 1 - P(0) = 1 - 0.0183 = 0.9817.$

(iii) $P(x = 3) = \frac{(4)^3 e^{-4}}{3!} = 0.1952$

(iv) $P(x \geq 3) = 1 - P(x < 3) = 1 - [P(0) + P(1) + P(2)]$

$= 1 - \left[\frac{(4)^0 e^{-4}}{0!} + \frac{(4)^1 e^{-4}}{1!} + \frac{(4)^2 e^{-4}}{2!}\right] = 0.762.$

(v) $P(2 \leq x \leq 4) = P(2 \text{ to } 4 \text{ defective fusses}) = P(2) + P(3) + P(4)$

$= \frac{(4)^2 e^{-4}}{2!} + \frac{(4)^3 e^{-4}}{3!} + \frac{(4)^4 e^{-4}}{4!} = 0.5368.$

**6. A certain screw making machine produces an average of 2 defective screws out of 100, and packs them in boxes of 500. Find the probability that a box contains 15 defective screws.**

**Solution:**

Given $\quad p = \frac{2}{100} = 0.02, \quad n = 500 \therefore \quad \mu = np = 500 \times 0.02 = 10$

Poisson distribution is given by $\quad P(x) = \frac{\mu^x e^{-\mu}}{x!} = \frac{(10)^x e^{-10}}{x!}, \quad x = 0, 1, 2, \ldots$

$\therefore \quad P(15) = \frac{(10)^{15} e^{-10}}{15!} = 0.035.$

**7. Alpha particles are emitted by a radioactive source at an average rate of 5 in a 20**

**minutes interval. Using poison distribution, find the probability that there will be (i) exactly two emissions and (ii) at least two emissions, in a randomly chosen 20 minutes interval.**

**Solution :**

Given $\mu = 5$. Poisson distribution is given by $P(x) = \frac{\mu^x e^{-\mu}}{x!} = \frac{(5)^x e^{-5}}{x!}$, $x = 0, 1, 2, \ldots$

(i) $P(x = 2) = \frac{(5)^2 e^{-5}}{2!} = 0.0842.$

(ii) $P(x \geq 2) = 1 - P(x < 2) = 1 - [P(0) + P(1)]$

$$= 1 - \left[\frac{(5)^0 e^{-5}}{0!} + \frac{(5)^1 e^{-5}}{1!}\right] = 0.9596.$$

**8. It is known that 5% of books found at a certain bindery have defective bindings. Find the probability that 2 out of 100 books bind by this bindery will have defective bindings, using Poisson distribution.**

**Solution:**

Given$p = \frac{5}{100} = 0.05$, $n = 100$. $\therefore\ \mu = np = 100 \times 0.05 = 5$

Poisson distribution is given by $P(x) = \frac{\mu^x e^{-\mu}}{x!} = \frac{(5)^x e^{-5}}{x!}$, $x = 0, 1, 2, \ldots$

$\therefore\quad P(2) = \frac{(5)^2 e^{-5}}{2!} = 0.084.$

**HOMEWORK:**

1. A communication channel receives independent pulses at the rate of 12 pulses per micro second. The probability of transmission error is 0.001 for each micro second. Compute the probabilities of (i) no error during a micro second (ii) one error per micro second (iii) at most two errors.

**2. The no. of Gamma Rays emitted by a certain radioactive substance is a random variable having Poisson's distribution with mean is 5.8. If recording instrument is inoperative for more than 12 rays per second. What is the probability that it becomes inoperative? during any given second.**

**Continuous probability Distribution:**

If a random variable X takes every value in an interval, then it gives rise to continuous distribution of X.

The distribution defined by the variates like heights or weights are continuous distributions. Let X be a continuous random variable on a sample space S and the set $\{a \leq X \leq b\}$ is an event in S. Then the probability $P(a \leq X \leq b)$ is well defined and is given by

$P(a \leq X \leq b) = \int_a^b f(x)\, dx$ [where $f(x)$ is a function defined in S].

The function $f(x)$ is called the *probability density function* (p. d. f) of X or *continuous probability distribution*of X if it satisfies (i) $f(x) \geq 0, \quad \forall\, x$ and (ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$. The continuous curve $f(x)$ is called the probability curve.

## Distribution function:

If $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\, dx$, then $F(x)$ is called the cumulative distribution function (c.d. f) or simply the distribution function of the continuous variate X. It is the probability that the value of the variate X will be $\leq x$.

The distribution function $F(x)$ has the following properties:

(i) $F'(x) = f(x) \geq 0$, so that $F(x)$ is a non-decreasing function,

(ii) $F(-\infty) = 0$, (iii) $F(\infty) = 1$,

(iv) $P(a \leq X \leq b) = \int_a^b f(x)\, dx = \int_{-\infty}^{b} f(x)\, dx - \int_{-\infty}^{a} f(x)\, dx = F(b) - F(a)$.

**Expectation:**

The mean value of the probability distribution of a variate X is the expectation and is denoted by E[x]. $\therefore\ \mu = E[x] = \int_{-\infty}^{\infty} x\, f(x)\, dx$ and the variance of the distribution is

$\text{var}(x) = \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2$.

i.e., $\text{var}(x) = E[x^2] - \{E[x]\}^2$, where $E[x^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx$.

Standard deviation is $\sigma = \sqrt{\text{var}(x)}$.

## Problems:

**1. Is the function defined as follows a density function?**

$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ **If so, (i) determine the probability that the variate having this will fall in the interval (1, 2)? (ii) Also, find the cumulative probability function $F(2)$?**

**Solution:**

Clearly, $f(x) \geq 0, \quad \forall\ x$ and

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty} e^{-x} dx = [-e^{-x}]_0^{\infty} = -(0-1) = 1.$$

$\therefore \quad f(x)$ is a density function (p. d. f)?

(i) $P(1 \leq x \leq 2) = \int_1^2 f(x)\,dx = \int_1^2 e^{-x}\,dx = [-e^{-x}]_1^2 = -[e^{-2} - e^{-1}] = 0.2325.$

(ii)$F(x) = \int_{-\infty}^{x} f(x)\,dx \Rightarrow F(2) = \int_{-\infty}^{2} f(x)\,dx = \int_0^2 e^{-x}\,dx = [-e^{-x}]_0^2 = 0.8646.$

**2. If X is a continuous random variable with probability density function given by**

$$f(x) = \begin{cases} kx, & 0 \leq x < 2 \\ 2k, & 2 \leq x < 4 \\ -kx + 6k, & 4 \leq x < 6 \end{cases}$$

**Find $k$ and mean value of X.**

**Solution:**

(i) we have $f(x) \geq 0 k > 0$ and $\int_{-\infty}^{\infty} f(x)\,dx = 1.$

$\therefore \int_{-\infty}^{0} 0\,dx + \int_0^2 kx\,dx + \int_2^4 2k\,dx + \int_4^6 (-kx + 6k)\,dx + \int_6^{\infty} 0\,dx = 1.$

$\therefore k\left[\frac{x^2}{2}\right]_0^2 + 2k[x]_2^4 - k\left[\frac{x^2}{2}\right]_4^6 + 6k[x]_4^6 = 1.$

$\therefore \quad \frac{k}{2}[4-0] + 2k[4-2] - \frac{k}{2}[36-16] + 6k[6-4] = 1 \therefore \quad 8k = 1.$

$$\therefore \quad k = \frac{1}{8}$$

(ii) Mean = $\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx = \int_0^2 kx^2\,dx + \int_2^4 2kx\,dx + \int_4^6 (-kx^2 + 6kx)\,dx.$

$$\therefore \ \mu = k\left[\frac{x^3}{3}\right]_0^2 + 2k\left[\frac{x^2}{2}\right]_2^4 - k\left[\frac{x^3}{3}\right]_4^6 + 6k\left[\frac{x^2}{2}\right]_4^6$$

$\therefore \ \mu = \frac{1}{8}\left[\frac{8}{3} - 0\right] + \frac{1}{8}[16-4] - \frac{1}{8}\left[\frac{216}{3} - \frac{4}{3}\right] + \frac{3}{8}[36-16] = 3.$

**3. A function is defined as follows:**

$$f(x) = \begin{cases} 0, & x < 2 \\ \frac{1}{18}(2x+3), & 2 \leq x < 4 \\ 0, & x > 4 \end{cases}$$

**Show that it is a density function. Find the probability that a variate having this density**

**will fall in the interval $2 \le x \le 3$ ?**

**Solution:**

(i) We note that $f(x) \ge 0$ and

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{2} 0\,dx + \int_{2}^{4} \frac{1}{18}(2x+3)dx + \int_{4}^{\infty} 0\,dx = \frac{1}{18}\left[\frac{2x^2}{2} + 3x\right]_2^4$$

$= \frac{1}{18}[(16-4) + 3(4-2)] = 1.$

$\therefore$ $f(x)$ is a p. d. f.

(ii) $P(2 \le x \le 3) = \int_2^3 f(x)\,dx = \int_2^3 \frac{1}{18}(2x+3)\,dx = \frac{1}{18}\left[2.\frac{x^2}{2} + 3x\right]_2^3 = 0.4444.$

**4. A random variable $x$ has the density function $P(x) = \begin{cases} kx^2, & 0 \le x \le 3 \\ 0, & else\ where \end{cases}$**

**Evaluate $k$, and find (i) $P(x \le 1)$ (ii) $P(1 \le x \le 2)$ (iii) $P(x \le 2)$ (iv) $P(x > 1)$ (v) $P(x > 2)$. Also, determine the mean, variance, and standard deviation of the distribution.**

**Solution:**

We have, $P(x) \ge 0$ and $\int_{-\infty}^{\infty} P(x)\,dx = 1 \therefore k > 0$ and $\int_0^3 kx^2\,dx = 1 \therefore k\left[\frac{x^3}{3}\right]_0^3 = 1 \therefore$ $9k = 1. \therefore \quad k = \frac{1}{9}$.

(i) $P(x \le 1) = \int_{-\infty}^{1} P(x)\,dx = \int_0^1 kx^2\,dx = \frac{1}{9}\left(\frac{x^3}{3}\right)_0^1 = \frac{1}{27}$.

(ii)$P(1 \le x \le 2) = \int_1^2 P(x)\,dx = \int_1^2 kx^2\,dx = \frac{7}{27}$.

(iii)$P(x \le 2) = \int_{-\infty}^{2} P(x)\,dx = \int_0^2 kx^2\,dx = \frac{8}{27}$.

(iv)$P(x > 1) = \int_1^{\infty} P(x)\,dx = \int_1^3 kx^2\,dx = \frac{26}{27}$.

(v) $P(x > 2) = \int_2^{\infty} P(x)\,dx = \int_2^3 kx^2\,dx = \frac{19}{27}$.

Mean $= \int_{-\infty}^{\infty} x\,P(x)\,dx = \int_0^3 kx^3\,dx = k\left[\frac{x^4}{4}\right]_0^3 = \frac{9}{4}$.

$\text{var(x)} = \text{V} = \sigma^2 = \int_{-\infty}^{\infty} x^2\,P(x)\,dx - \mu^2 = \int_0^3 kx^4\,dx - \mu^2 = k\left[\frac{x^5}{5}\right]_0^3 - \left(\frac{9}{4}\right)^2 = \frac{27}{80}$.

$\therefore$ $\text{S.D} = \sigma = \sqrt{\text{V}} = \sqrt{\frac{27}{80}} = 0.5809.$

**5. The probability density $P(x)$ of a continuous random variable is given by**

**$P(x) = y_0 e^{-|x|}, -\infty < x < \infty$. Prove that $y_0 = \frac{1}{2}$. Find the mean and variance of the distribution.**

**Solution:**

We have, $P(x) \geq 0$ and $\int_{-\infty}^{\infty} P(x)\,dx = 1. \therefore\ y_0 > 0$ and

$\because\ |x| = -x \text{ if } x \leq 0$
$= x \text{ if } x \geq 0$

$\int_{-\infty}^{\infty} y_0 e^{-|x|}\,dx = 1.$ But $e^{-|x|}$ is even. $\therefore\ 2\int_0^{\infty} y_0 e^{-x}\,dx = 1$

$\therefore\ 2y_0 \left[\frac{e^{-x}}{-1}\right]_0^{\infty} = 1. \therefore\ -2y_0[0-1] = 1. \therefore\ y_0 = \frac{1}{2}.$

(i) Mean = $\mu = \int_{-\infty}^{\infty} x\,P(x)\,dx = y_0 \int_{-\infty}^{\infty} xe^{-|x|}\,dx = y_0\,.[0] = 0. \because\ e^{-|x|}$ is odd

(ii) var(x) = V = $\sigma^2 = \int_{-\infty}^{\infty} x^2\,P(x)\,dx - \mu^2 = y_0 \int_{-\infty}^{\infty} x^2 e^{-|x|}\,dx - 0$

$= \frac{1}{2} \times 2\int_0^{\infty} x^2 e^{-x}\,dx \because\ x^2 e^{-|x|}$ is even

$$\text{var(x)} = \left[x^2\left(\frac{e^{-x}}{-1}\right) - 2x\left(\frac{e^{-x}}{1}\right) + 2\left(\frac{e^{-x}}{-1}\right)\right]_0^{\infty} = 2.$$

**6. Find $k$such that $f(x)=\begin{cases} kxe^{-x}, & 0 < x < 1 \\ 0, & otherwise \end{cases}$ is a probability distribution function. Find the mean.**

**Solution:**

we have $f(x) \geq 0 k > 0$ and $\int_{-\infty}^{\infty} f(x)\,dx = 1.$

$k > 0$ and $\int_0^1 kx\,e^{-x}\,dx = 1$Applying Bernoulli's rule we have,

$$k[x(e^{-x}) - 1.(e^{-x})]_0^1 = 1$$

$$k\left[-\frac{1}{e} - \left(\frac{1}{e} - 1\right)\right] = 1 \ ,\quad k\left(1 - \frac{2}{e}\right) = 1 \ ,\ k = \frac{e}{e-2}$$

Mean = $\mu = \int_{-\infty}^{\infty} x\,f(x)\,dx = \int_0^1 x.\frac{e}{e-2} xe^{-x} dx = \frac{e}{e-2}\int_0^1 x^2 e^{-x}\,dx$

$=\frac{e}{e-2}[x^2(-e^{-x}) - (2x)(e^{-x}) + 2(-e^{-x})]_0^1$

$= \frac{e}{e-2}\left[-\frac{1}{e} - \frac{2}{e} - 2\left(\frac{1}{e} - 1\right)\right] = \frac{e}{e-2}\left[2 - \frac{5}{e}\right]$

$\boldsymbol{\mu = \frac{2e-5}{e-2}}$

**NOTE:**

**Definite Integral Property:**

$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$ if $f(-x) = f(x)$ i.e $f(x)$ is even function

$\int_{-a}^{a} f(x)\,dx = 0$ if $f(-x) = -f(x)$ i.e $f(x)$ is odd function.

## HOME WORK:

**1. A random variable $x$ has the following density function** $f(x)=\begin{cases} kx^2, & -3 \le x \le 3 \\ 0, & otherwise \end{cases}$

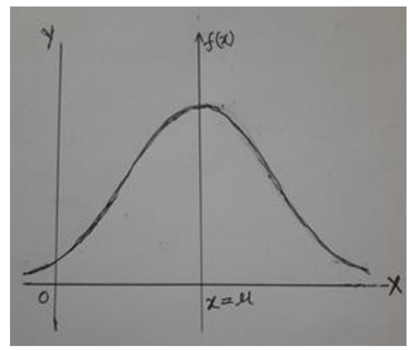
**Evaluate $k$ And find (i) $P(1 \le x \le 2)$ (ii) $P(x \le 2)$ (iii) $P(x > 1)$.**

**2.A random variable $x$ has the density function $f(x) = \frac{k}{1+x^2}$, $-\infty < x < \infty$ Determine $k$ and hence evaluate (i) $P(x \ge 0)$ (ii) $P(0 < x < 1)$.**

## Normal Distribution:

The continuous probability distribution having the p. d. f. given by

$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$ ------ (1)Where $-\infty < \mu < \infty$ and $\sigma > 0$ is called a normal distribution. [It is also a limiting case of binomial distribution.] $\mu$ is the mean and $\sigma$ is the standard deviation of the distribution?$x$ is called the normal variate. $f(x)$ is a bell shaped curve and symmetrical about the line? $x = \mu$. This curve is called a normal curve. Clearly,




(i) $f(x) \ge 0$ and (ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$.

The probability of $x$ lying between $x_1$ and $x_2$ is given by the area under the normal curve from $x_1$ and $x_2$. i.e. $P(x_1 \le x \le x_2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{x_1}^{x_2} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx$.------- (2)

Take $\frac{x-\mu}{\sigma} = z$. $\therefore$ $x - \mu = \sigma z. \therefore$ $dx = \sigma\, dz$.

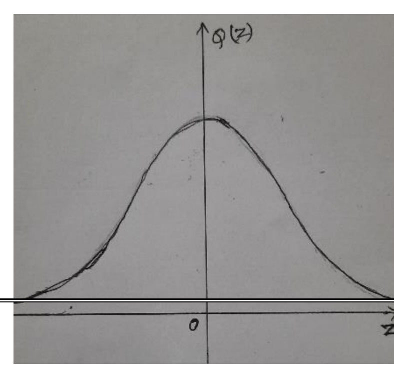
When $x = x_1$, take $z = z_1 = \frac{x_1-\mu}{\sigma}$ and when $x = x_2$, take $z = z_2 = \frac{x_2-\mu}{\sigma}$ .

$$\therefore \quad P(z_1 \le z \le z_2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{z_1}^{z_2} e^{\frac{-z^2}{2}} \sigma\, dz = \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{\frac{-z^2}{2}} dz.$$

$$\therefore \quad P(z_1 \le z \le z_2) = \frac{1}{\sqrt{2\pi}} \int_0^{z_2} e^{\frac{-z^2}{2}} dz - \frac{1}{\sqrt{2\pi}} \int_0^{z_1} e^{\frac{-z^2}{2}} dz = \int_0^{z_2} \varphi(z)\, dz - \int_0^{z_1} \varphi(z)\, dz.$$

$$\therefore \quad P(z_1 \le z \le z_2) = P(0 \le z \le z_2) - P(0 \le z \le z_1) = P(z_2) - P(z_1).$$

This gives the area under the curve $\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}$ between $z = z_1$ and $z = z_2$. This curve is called the standard normal curve, which is



symmetrical about the line $z = 0$. $z$ is called the standard normal variate.

It can be obtained by taking $\mu = 0$ and $\sigma = 1$ in (1).

In particular, we write,

$P(z) = \int_0^z \varphi(z)\, dz = \frac{1}{\sqrt{2\pi}} \int_0^z e^{\frac{-z^2}{2}}\, dz = A(z)$ for various values of $z$.

**Applications of normal distribution:**

This distribution is applied for
(i) Calculation of errors made by chance in experimental measurements.
(ii) Computation of hit probability of a shot.
(iii) Statistical inference in almost every branch of science.

**Note:**
The value of $P(z)$ or $A(z)$ are given in a table which is known as normal distribution table.

(1) $P(-\infty < z < \infty) = 1$

(2)$P(-\infty < z \le 0) = 0.5$ i.e $P(z \le 0) = 0.5$

(3)$P(0 \le z < \infty) = 0.5$ i.e $P(z \ge 0) = 0.5$

(4) $P(-\infty < z \le z_1) = P(-\infty < z \le 0) + P(0 \le z < z_1) = P(z \le 0) + P(z_1) = 0.5 + P(z_1)$

i.e., $P(z \le z_1) = 0.5 + P(z_1)$

(5) $P(z_2 \le z < \infty) = P(0 \le z < \infty) - P(0 \le z \le z_2) = 0.5 - P(z_2)$.

**Calculator operation for normal distribution:**

(i) MODEL: f x 991MS – mode – SD, shift – 3 (Distribution), substitute for Q.
(ii) MODEL: f x 991ES – mode – STAT (3), AC– shift – 1– 5 (Distribution), substitute for Q.

**Problems:**

**1. If X is a normal variate with mean 30 and S.D. 5, find the probabilities that (i) $26 \le X \le 40$ (ii) $X \ge 45$ (iii) $|X - 30| > 5$.**

**Solution:**
Given $\mu = 30$, $\sigma = 5$.∴Standard normal variate is$z = \frac{x-\mu}{\sigma}$ ∴ $z = \frac{x-30}{5}$.

(i) $P(26 \le X \le 40) = P\left(\frac{26-30}{5} \le z \le \frac{40-30}{5}\right) = P(-0.8 \le z \le 2)$.

$= P(-0.8 \le z \le 0) + P(0 \le z \le 2) = P(0 \le z \le 0.8) + P(2).$

$= P(0.8) + P(2) = 0.2881 + 0.4772 = 0.7653.$

(ii) $P(X \ge 45) = P\left(z \ge \frac{45-30}{5}\right) = P(z \ge 3) = P(z \ge 0) - P(0 \le z \le 3)$

$= 0.5 - P(3) = 0.5 - 0.4986 = 0.0014$

(iii) $P(|X - 30| > 5) = 1 - P(|X - 30| \le 5) = 1 - P(-5 \le X - 30 \le 5)$

$= 1 - P(25 \le X \le 35) = 1 - P(-1 \le X \le 1)$

$= 1 - 2P(1) = 1 - 0.6826 = 0.3174.$

**2. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. Given A (0.5) =0.19 and A (1.4) =0.42, where A(z) is the area under the standard normal curve from 0 to z.**

**Solution:**

Let $x$ be the normal variate and z be the standard normal variate, then $z = \frac{x-\mu}{\sigma}$, where $\mu$ is the mean and $\sigma$ is the S.D.

Given $P(x < 45) = \frac{31}{100} = 0.31$ and $P(x > 64) = \frac{8}{100} = 0.08.$

$\therefore$ $P\left(z < \frac{45-\mu}{\sigma}\right) = 0.31$ and $P\left(z > \frac{64-\mu}{\sigma}\right) = 0.08.$

i.e., $P\left(z > \frac{\mu-45}{\sigma}\right) = 0.31$

$\therefore$ $0.5 - A\left(\frac{\mu-45}{\sigma}\right) = 0.31$ and $0.5 - A\left(\frac{64-\mu}{\sigma}\right) = 0.08.$

$\therefore$ $0.19 = A\left(\frac{\mu-45}{\sigma}\right)$ and $0.42 = A\left(\frac{64-\mu}{\sigma}\right).$

$\therefore$ $A(0.5) = A\left(\frac{\mu-45}{\sigma}\right)$ and $A(1.4) = A\left(\frac{64-\mu}{\sigma}\right).$

$\therefore$ $0.5 = \frac{\mu-45}{\sigma}$ and $1.4 = \frac{64-\mu}{\sigma}.$

$\therefore$ $0.5\,\sigma = \mu - 45$ and $1.4\,\sigma = 64 - \mu.$

$\therefore$ $\mu - 0.5\,\sigma = 45$ and $\mu + 1.4\,\sigma = 64.$

Solving we get, $\mu = 50$ and $\sigma = 10.$

**3. In a normal distribution, 7% are under 35 and 89% are under 63. Find the mean and**

**standard deviation given that A (1.23) =0.39 and A (1.48) =0.43 in the usual notation.**

**Solution:**

Let $x$ be the normal variate and z be the standard normal variate, then $z = \frac{x-\mu}{\sigma}$, where $\mu$ is the mean and $\sigma$ is the S.D.

Given $P(x < 35) = 0.07$ and $P(x < 63) = 0.89$.

$\therefore$ $P\left(z < \frac{35-\mu}{\sigma}\right) = 0.07$ and $P\left(z < \frac{63-\mu}{\sigma}\right) = 0.89$.

i.e., $P\left(z > \frac{\mu-35}{\sigma}\right) = 0.07$

$\therefore$ $0.5 - A\left(\frac{\mu-35}{\sigma}\right) = 0.07$ and $0.5 + A\left(\frac{63-\mu}{\sigma}\right) = 0.89$.

$\therefore$ $A\left(\frac{\mu-35}{\sigma}\right) = 0.43 = A(1.48)$ and $A\left(\frac{63-\mu}{\sigma}\right) = 0.39 = A(1.23)$

$\therefore$ $\frac{\mu-35}{\sigma} = 1.48$ and $\frac{63-\mu}{\sigma} = 0.39$.

$\therefore$ $1.48\,\sigma = \mu - 35$ and $0.39\,\sigma = 63 - \mu$.

$\therefore$ $\mu - 1.48\,\sigma = 35$ and $\mu + 0.29\,\sigma = 63$.

Solving we get, $\mu = 50.29$ and $\sigma = 10.33$.

**4. In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for (i) more than 2150 hours (ii) Less than 1950 hours and (iii) more than 1920 hours and but less than 2160 hours.**

**Solution:**

Given$\mu = 2040$, $\sigma = 60$. $\therefore$The standard normal variate is $z = \frac{x-\mu}{\sigma} = \frac{x-2040}{60}$.

(i) P(A bulb burn for more than 2150 hours) = $P(x > 2150) = P\left(z > \frac{2150-2040}{60}\right)$

$= P(z > 1.83) = 0.5 - P(1.83) = 0.5 - 0.4664 = 0.0336.$

$\therefore$ Number of bulbs burn for more than 2150 hours $= 0.0336 \times 2000 \approx 67$.

(ii) P(A bulb burn for less than 1950 hours)= $P(x < 1950) = P\left(z < \frac{1950-2040}{60}\right)$

$= P(z < -1.5) = P(z > 1.5) = 0.5 - P(1.5) = 0.5 - 0.4332 = 0.0668.$

∴ Number of bulbs burn for less than 1950 hours= $0.0668 \times 2000 \approx 134$.

(iii) P (A bulb burn for more than 1920 hours and but less than 2160 hours)

$= P(1920 < x < 2160) = P(-2 < z < 2) = 2P(2)$

$= 2(0.4772) = 0.9544$.

∴ Number of bulbs burn between 1920 hours and 2160 hours= $0.9544 \times 2000 \approx 1909$.

**5. The mean weight of 500 students at a certain school is 50 kgs and S.D. is 6 kgs. Assuming that the weights are normally distributed, find the expected number of students weighing (i) between 40 and 50 kgs and (ii) more than 60 kgs.**

**Solution:**

Given$\mu = 50$, $\sigma = 6$. ∴The standard normal variate is $z = \frac{x-\mu}{\sigma} = \frac{x-50}{6}$.

(i) $P(40 < x < 50) = P(-1.6667 < z < 0) = P(0 < z < 1.6667) = P(1.6667)$

$= 0.4525$.

∴ Number of students weighing between 40 and 50 kgs $= 0.4525 \times 500 = 226$.

(ii) $P(x > 60) = P(z > 1.6667) = 0.5 - P(1.6667) = 0.5 - 0.4525 = 0.0478$.

∴ Number of students weighingmore than 60 kgs $= 0.0478 \times 500 = 24$.

**6. The daily wages of 1000 workmen are normally distributed around a mean of Rs. 70 and standard deviation of Rs. 5. Estimate the number of workers whose daily wages will be (i) less than Rs. 65 (ii) between Rs. 65 and Rs. 75 (iii) more than Rs. 75.**

**Solution:**

Given$\mu = 70$, $\sigma = 5$. ∴The standard normal variate is $z = \frac{x-\mu}{\sigma} = \frac{x-70}{5}$.

(i) $P(x < 65) = P(z < -1) = P(z > 1) = 0.5 - P(1) = 0.5 - 0.3415 = 0.1587$.

∴ Number of workers whose daily wages will be less than Rs. 65 $= 0.1587 \times 1000 \approx 159$.

(ii) $P(65 < x < 75) = P(-1 < z < 1) = 2P(1) = 0.6826$.

∴ Number of workers whose daily wages will be between Rs.65 and Rs.75= $0.6826\ x 1000$.

$\approx 683$.

(iii) $P(x > 75) = P(z > 1) = 0.5 - P(1) = 0.1587$

∴ Number of workers whose daily wages will be more than Rs.75 $= 0.1587 \times 1000 \approx 159$.

**HOMEWORK:**

**1. The mean height of 500 students is 151 cm and the standard deviation is 15cm.Assuming that the heights are normally distributed, find how many students heights lie between 120 and 155 cm.**

**2. Assuming the mean height of soldiers to be 68.22 inches with a variance of 10.8 inches.How many soldiers in a regiment of 1000 would you expect to be (i)Over six feet? tall and (ii) below 5.5 feet?Assuming heights to be normally distributed.**