Free Electron concept, Electrical conductivity in metals, Resistivity and Mobility, Concept of Phonon, Matthiessen's rule. Introduction to Super Conductors, Temperature dependence of resistivity, Meissner's Effect, Silsbee Effect, Types of Superconductors, Temperature dependence of critical field, BCS theory (Qualitative), Quantum Tunneling, High- Temperature superconductivity, Josephson Junction, DC and AC SQUIDs (Qualitative), MAGLEV, Applications in Quantum Computing (Mention). Numerical problems.

Free Electron concept:

Classical Free electron theory (Drude-Lorentz Theory):

In metals valence electrons are responsible for conduction. In case of copper, there is one valence electron. The core electron along with the valence electron balances the positive charge on the nucleus. Thus, atom is neutral.

When we consider large number of atoms, the boundaries of the neighboring atoms overlap on each other. The valence electrons can move from one atom to other atom because of overlapping of atoms. These electrons are called free electrons which are responsible for conduction.

When an electron comes out of the parent atom, the atom becomes a positive ion. These ionic cores are called lattice ions. These lattice ions even though are localized they will be vibrating in their mean position. These vibrations are due to thermal agitation. Due to the thermal energy the free electrons keep on moving inside the conductor. During their motion they collide with vibrating lattice ions. After collision they move in random directions. Due to this, the average velocity acquired by the electron is zero. Thus, no current in the conductor in the absence of field.

When electric field is applied, there will be overall shift in the position of electron and the electrons slowly drifted in the direction opposite to that of applied field. The displacement per unit time is called drift velocity which is a constant for free electron in steady state.

The velocity of electron in steady state in an applied electric field is called drift velocity. It is denoted by $V_{d.}$

Assumptions of Classical free electron theory:

1. A metal is imagined as a three-dimensional array of ions in between which there are freely moving valence electrons. The electric current in a metal is due to drift velocity of electrons in a direction opposite to the direction of applied field.
2. The free electrons are treated as equivalent to gas molecules and thus assumed to obey the laws of kinetic theory of gases. In the absence of the field, the energy associated with each electron at a temperature T will be (3/2)kT according to kinetic theory of gases, therefore $(3/2)kT=(1/2)mv^2$. Where 'k' is Boltzmann constant, 'T' is temperature and 'v' is thermal velocity.
3. The electric potential due to ionic cores is taken to be essentially constant throughout the material.
4. The attraction between the free electron and lattice ions and the repulsion between electron themselves are considered to be insignificant.

<u>Expression for electrical conductivity in metals:</u>

Consider motion of a free electron in a conductor in the influence of external electric field. If 'e' is a charge on electron and 'E' is the strength of applied field, then the force acting on the electron is given by,

$$F = eE \longrightarrow (1)$$

If 'm' is mass of electron, then as per Newton's second law of motion force acting on the electron is given by,

$$F=ma$$

$$F = m.\frac{dv}{dt} \longrightarrow (2)$$

From equation (1) and (2)

$$e\,E = m\,\frac{dv}{dt}$$

$$dv = \frac{e\,E\,dt}{m}$$

Integrating on both sides, $\int dv = \int_0^t \left(\frac{e\,E\,dt}{m}\right)$

Or $v = \frac{e\,E\,t}{m}$ Where, 't' time of traverse

If $t = \tau$, is taken to be collision time and v is taken to be average velocity,

$$v = \frac{e\,E\,\tau}{m} \longrightarrow (3)$$

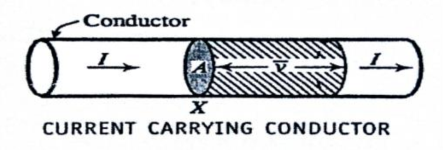
Electrical conductivity is given by, $\sigma = \frac{J}{E} \longrightarrow (4)$

Where J is current density, $J = \frac{I}{A} \longrightarrow (5)$

Where I is current in the conductor, A is area of cross section of the conductor

Therefore eqn (4) can be written as,

$$\sigma = \frac{I}{A\,E} \longrightarrow (6)$$



Consider a conductor, let 'I' be the current carrying in it, 'A' be the area of cross section, if 'v' is the velocity of conduction of electrons, then the distance covered by the electrons in unit time is numerically equal to velocity. These electrons swipe volume 'Av' in a unit time. If e is charge on the electron, 'n' is the electrons per unit volume then, the quantity of charge crossing a given point in a conductor per unit time is given by

$$I = nevA \longrightarrow (7)$$

Substituting eqn (7) in eqn (6), we get

$$\sigma = \frac{nev\,A}{A\,E}$$

$$\sigma = \frac{nev}{E} \longrightarrow (8)$$

Now by substituting eqn (3) in eqn (8), we get

$$\boldsymbol{\sigma = \frac{ne^2\tau}{m}}$$ $\Omega^{-1}m^{-1}$, This is the expression for electrical conductivity.

The unit of conductivity is Sm^{-1} (siemens per metre).

Resistivity:

The **resistance** of a piece of material depends on its resistivity and also its size and shape. The resistance R of a wire with cross sectional area A and length L (**Figure 1**), made from a material of resistivity ρ ('rho') is given by the equation R,

$$\rho = AR/L$$

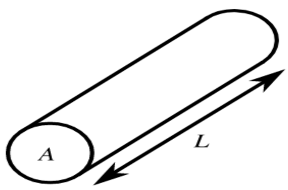


Figure 1: Diagram showing a section of wire, with cross sectional area A and length L.

Resistivity is a property that describes the extent to which a material opposes the flow of electric current through it. It is a property of the material itself (not the size or shape of the sample), usually depends on temperature and may depend on other quantities such as pressure.

Electrical conductivity σ (sigma) is the reciprocal of resistivity and can be written as

$$\sigma = 1/\rho$$

$$\rho = \frac{m}{ne^2\tau}$$ The unit of resistivity is the Ωm (ohm metre).

Mobility:

Electrical mobility is the ability of charged particles (such as electrons or protons) to move through a medium in response to an electric field that is pulling them.
When an electric field is applied across a piece of material, the electrons respond by moving with an average velocity called the drift velocity. Then the electron mobility μ is defined as ratio of drift velocity to the applied electric field.

$\mu = \frac{V_d}{E}$ **unit is m^2/Vs.**

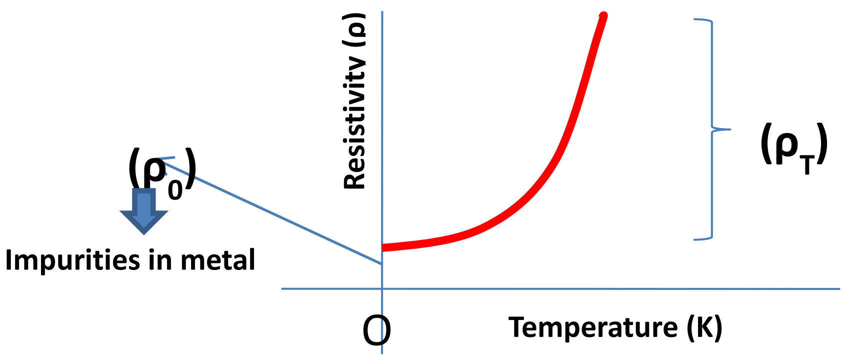
Concept of Phonon:

- A phonon can be defined as a discrete unit of vibrational mechanical energy, the phonons exist with a discrete amount of energy given by $E=\hbar\omega$.
- Vibration of lattice gives phonon.
- A phonon is the quantum energy of the lattice vibration, just like photons are the quantum energy of electromagnetic radiations. The energy of each phonon is given by: ⇒ E = hγ
- Phonon were suggested for the quantum lattice vibrational energy by Frenkel in 1935. Thus, phonons are the quanta of sound just like a photon is a quanta or the packet of energy for electromagnetic waves.

Matthiessen's rule:

Effect of Temperature and Impurities on the electrical conductivity in metals

Metals have loosely bound electrons in their outermost shell or valence shell which is responsible for conduction. These electrons are called free electrons/conduction electrons.
The dependence of resistivity ( ) on temperature is as shown below.



Matthiessen's rule : "The total resistivity of a metal is the sum of the resistivity due to impurities(ρ_o) at T=0K and the resistivity due to phonon scattering which is temperature dependent (ρ_T)". We can see that the resistance decreases with temperature and reaches minimum at T=0K .

The Variation is expressed by the Matthiessen's rule

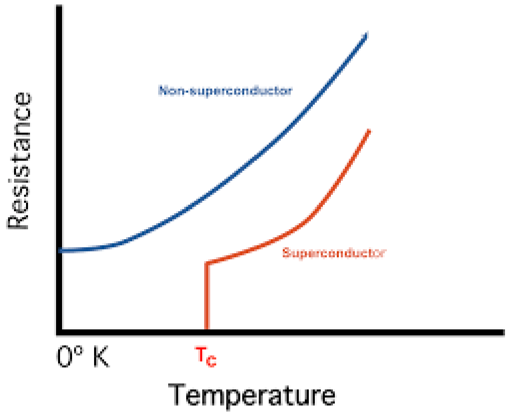
$$= \rho_o + \rho_{(T)}$$

Where, ρ →the resistivity of the given metal
ρ_o → the residual resistivity
$_{(T)}$→ the temperature dependent

Introduction to Super Conductors:

Certain metals, alloys and compounds exhibit zero resistivity and hence infinite conductivity at a temperature above 0 K. These materials are called as superconductors and the phenomenon is known as Superconductivity. It was first observed by the Dutch physicist Kammerlingh Onnes in 1911.

"The resistance offered by certain materials to the flow of electric current abruptly drops to zero below a threshold temperature. This phenomenon is called superconductivity and the threshold temperature is called critical temperature".



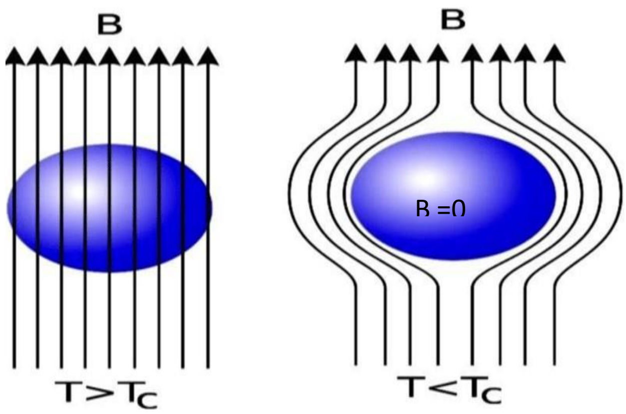
Critical Temperature: Temperature at which the resistivity of the material drops to zero is called as Critical Temperature (Tc) or Transition temperature

Ex: Hg = 4.2 K, Pb = 7.2 K, Nb = 4.5 K, Yitrium Barium Copper Oxide = 92 K etc

Meisner Effect (Effect of Magnetic field):

"A superconducting material kept in the magnetic field expels the magnetic flux out of its body when cooled below the critical temperature and thus becomes a perfect diamagnet. This effect is called Meissner effect".

Thus, the super conductor shows perfect diamagnetism.



We have the magnetic field inside the specimen,

$B = \mu_0 (H + M)$

H = Applied magnetic field and

M = Magnetization in the specimen.

According to Meisner effect, when the specimen is in superconducting state, B = 0

Thus H = -M,

Then, Magnetic susceptibility = (M/H) = -1.

This is the indication for a perfect diamagnetic material.

<u>Critical field OR Critical Magnetic field:</u>

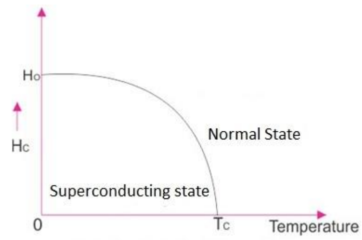
It is defined as the magnetic field required for switching the material from superconducting state to normal state and is denoted by H_C. When once the applied magnetic field is removed, the material will regain its superconducting property, provided T < Tc

Thus, the material will remain in the superconducting state below H_C and above H_C the material will be in the normal state.

Temperature dependence of critical magnetic field H_C:

- Magnitude of H_C depends on temperature
- If H_0 the critical magnetic field at T = 0 K and H_C the critical magnetic field at T^0K,then

$$H_C = H_0\left[1 - \frac{T^2}{T_C^2}\right]$$



Variation of Critical magnetic field with temperature

It can be seen that

- The curve of H_C v/s T is almost parabolic
- If the superconductor is held at 0 K, then higher magnetic field is required to destroy the superconducting property
- When the temperature of the superconductor is close to T_C, then lesser magnetic field is sufficient to destroy the superconducting property.

<u>Silsbee effect:</u>

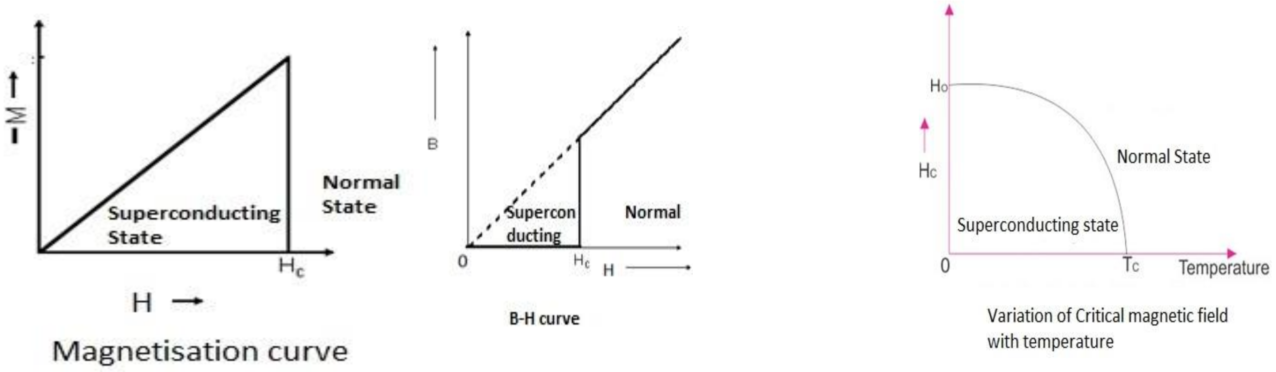
- The **Silsbee effect** or Silsbee current refers to the effect by which, if the current exceeds a critical level, the superconducting state will be destroyed.
- The ability of an electrical current to destroy superconductivity by means of the magnetic field generated by the current. The temperature of the material is not raised, and the effect is identical to external application of a magnetic field.

Types of Superconductors:

- The Classification is based on the response shown by the super conductors in the applied magnetic field.
- The response curve of magnetization **v/s** applied magnetic field show a different nature of variation for different category of superconductors.
- This classification provides useful information for the selection of superconductors in the development of high field magnets.

Type -1 superconductor (Soft superconductors):

- They exhibit complete Meisner effect.
- Material in the superconducting state retains its diamagnetic nature, until the critical field is reached.
- Once the critical field is reached, the material suddenly loses superconducting property.
- Here onwards the flux starts penetrating the specimen.
- These superconductors have low critical magnetic field.
- They are not useful for the construction of superconducting magnets. Ex: Hg, Pb, Nb, Sn etc

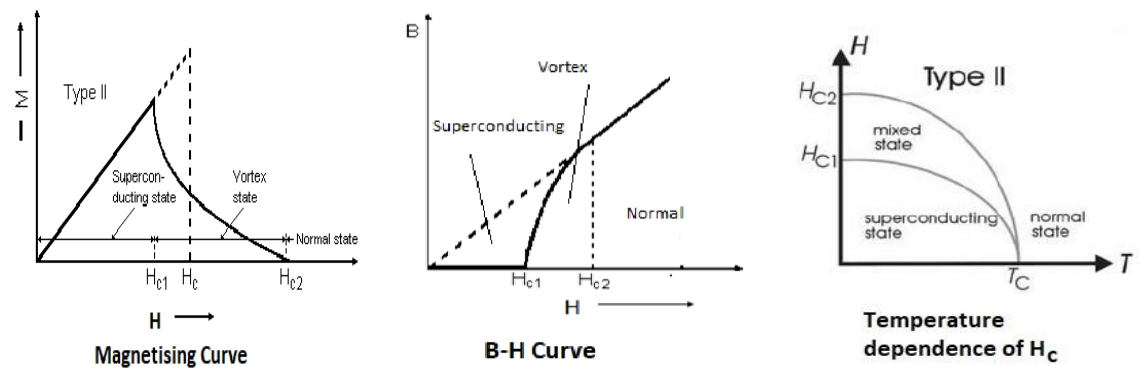


Response of Type 1 superconductor against applied magnetic field

Type -2 Superconductors (Hard Superconductors):

- They do not exhibit complete Meisner effect.
- They are characterized by two critical magnetic fields namely lower critical field H_{C1} and upper critical field H_{C2}.
- For the applied field less than H_{C1} the magnetic field is completed expelled out of the superconductor and it behaves as perfect diamagnetic.
- When the field is greater than H_{C1} and less than H_{C2}, the flux starts penetrating the specimen. In this region, the material is not perfect diamagnetic, but still retains zero resistance. Hence it is called as mixed state or vortex state.
- When the applied field exceeds H_{C2}, all the flux will penetrate the specimen and the material becomes a normal conductor.

  Ex : - Compounds like Y-Ba-Cu-O, Bi-Sr-Ca-Cu-O etc

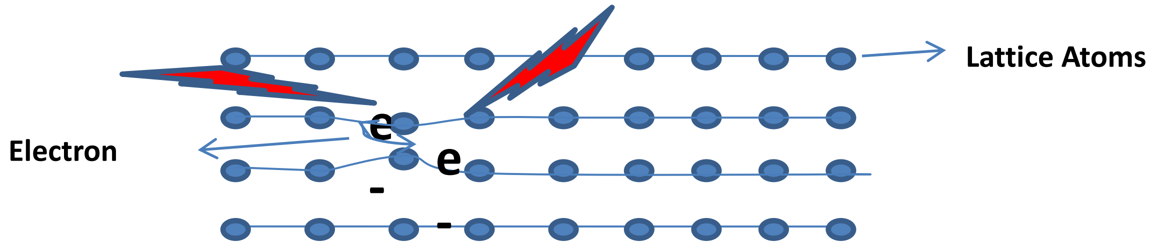


Response of Type2 superconductor against applied magnetic field

Type 1	Type 2
1. Exhibits complete Meisner effect.	1. Exhibits partial Meisner effect.
2. Only one Critical field.	2. Two critical fields namely lower and upper.
3. No vortex state is present.	3. Vortex state is present.
4. Low critical temp. (<10K).	4. High critical temp (>10K).
5. Low critical magnetic field.	5. High critical magnetic field.
6. Cannot be used to preparehigh field magnets.	6. Can be used to prepare high field magnets.

<u>BCS Theory: (Bardeen, Copper and Schrieffer Theory)</u>

- ✓ In 1957 BCS theory explains the phenomenon of Superconductivity.
- ✓ This theory is based upon the formation of cooper pairs, which is a quantum mechanical concept.



- ✓ When the free electron leaves the atom, the atom becomes positively charged. When electron comes near a positive ion core of the lattice, it experiences an attractive force because of the opposite polarity between electron and ion.
- ✓ Due to this attraction, the ion is displaced from its position, leading to lattice distortion. At the same time, if another electron which comes near that place will also interact with the distorted lattice. This process is looked upon as equivalent to interaction between the two electrons through the lattice. The process is called **"electron-lattice-electron"** interaction through the phonon field".
- ✓ The attractive force between two electrons will be maximum if they have equal and opposite spin and momentum. This force will even exceed the coulomb repulsive force between electrons. "**Hence, cooper pair is a bound pair of electrons formed by the interaction between the electron with opposite spin and momentum in a phonon field"**
- ✓ Each cooper pair is associated with a wavefunction. The wavefunction with similar cooper pairs start overlapping which may extend over 10^6 other pairs. This leads to a union of vast number of cooper pairs.
- ✓ When the electron flow in the form of cooper pairs, they do not encounter any scattering and the resistance factor vanishes or in other words conductivity becomes infinite hence a material becomes superconductor**.**

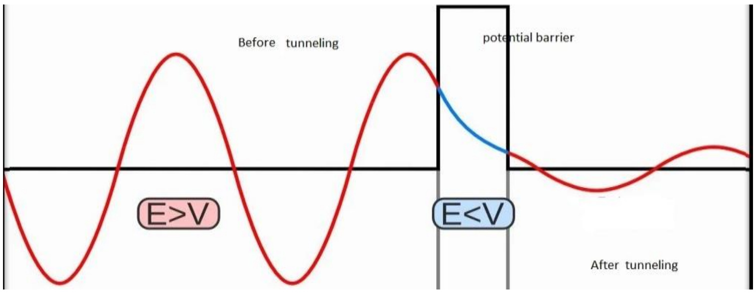
<u>High temperature superconductivity:</u>

- ✓ "Superconductors having higher critical temperatures are called high temperature superconductors".
- ✓ All high temperature superconductors are not pure metals, but they are different types of oxides of copper. And bear a particular type of crystal structure called Perovskite crystal structure.
- ✓ It is found that, the critical temperature is higher for those materials having more number of copper oxygen layers.
- ✓ It is found that the formations of super currents in high temperature superconductors are directional dependent.
- ✓ The super currents are strong in copper oxygen planes and weak in a direction perpendicular to the copper oxygen planes.

<u>Quantum Tunneling:</u>

- It is a quantum mechanical phenomenon in which an object such as an electron or atom passes through a potential energy barrier. This concept is not possible, according to classical mechanics.

- Tunneling is a outcome of wave nature of matter and is found in low mass particles like electrons, protons etc.
- Probability of transmission of a wave packet through a barrier decreases exponentially with the barrier height. When the quantum wave reaches the barrier, itsamplitude will decrease exponentially



(E the energy of the particle and V the potential energy of the barrier)

Josephson junction:

- A Josephson junction is made by sandwiching a thin layer of a non-superconducting material between two layers of superconducting material.
- If the barrier is an insulator, it must be about 30 angstroms thick or less. If the barrier is another metal, it can be as much as several nanometer thick.
- In this system, the cooper pairs tunnel through the barrier without resistance.
- This phenomenon of flow of current between two pieces of superconductor separated by a normal material is called as Josephson Effect and the current is called Josephson current. The current flows through the junction even in the absence of external DC voltage. Hence the Josephson current is present in the absence of supply voltage.
- If the external DC voltage is applied, current oscillates rapidly with a frequency of several GHz, leading to the development of AC voltage.

DC Josephson Effect:

- It is the phenomenon of flow of super current through the junction even in the absence of external emf. If the voltage across the junction is measured, it gives zero.
- Consider a Josephson junction containing two superconducting films separated by thin oxide layer. Here a cooper pair in the superconductor starts tunneling through the oxide layer which are represented by wave function.
- During this process the oxide layer introduces the phase difference between input and output wave functions.
- Due to this, super current flow through the junction, even in the absence of external source.
- The super current through the junction is

$$I_s = I_c \sin \varphi_0$$

I_c = critical current at zero voltage, which depends on the thickness of the junction layer and the temperature, φ_0 = Phase difference between the wave functions of cooper pairs

AC Josephson Effect:

- When dc voltage is applied across the Josephson junction, it leads to the development of oscillating current. In other words, an alternating emf of high frequency is established across the junction. This effect is called as AC Josephson effect.
- The oscillating current is because of the fact that, the application of dc voltage across the junction causes the additional phase change for the cooper pairs
- The energy difference of cooper pairs on both sides is of the order of 2eV
- Thus, the current

$$I_s = I_c \sin(\varphi_0 + \Delta\varphi)$$

$\Delta\varphi$ = phase difference, I_c is the critical current

The frequency of the generated AC is

$$f = 2eV / h$$

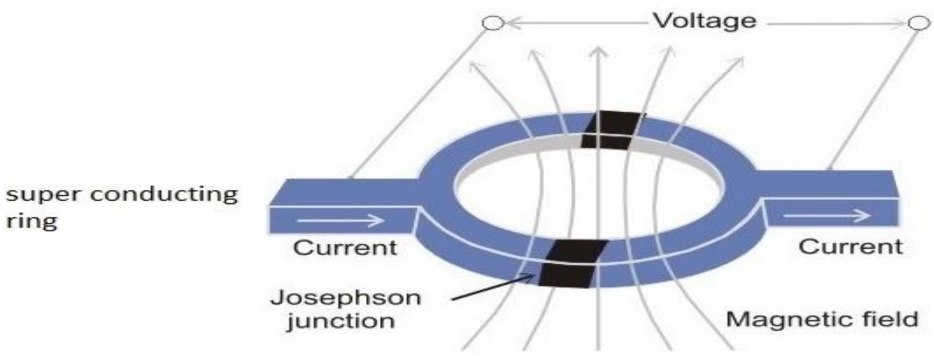
- Where 2eV is the energy difference between the cooper pairs on either side of the junction.

SQUIDS:

- Stands for Super conducting Quantum Interference Device.
- It is an instrument used to measure extremely weak magnetic field of the order 10^{-13}T.
- Hence it is a sensitive magnetometer.
- Heart of the SQUID is a super conducting ring containing one or more Josephsonjunctions.
- Two types of SQUIDS are available namely DC SQUID and RF SQUID.
- It works on the principle of Josephson effectDC SQUID.

It has two Josephson junctions connected in parallel and works on the interference of current from two junctions. It works on the principle of DC Josephson effect which is the phenomenon of flow of super current through the junction even in the absence of external emf.

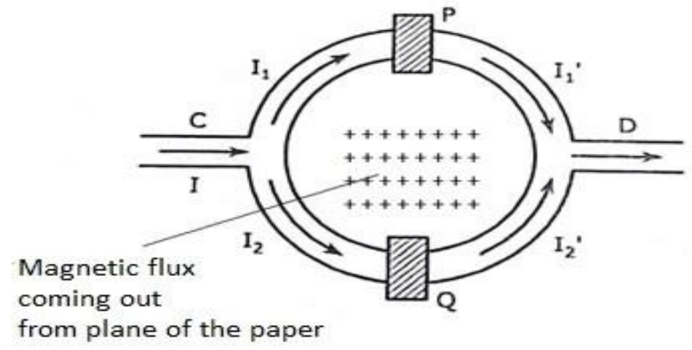
Construction and Working:



The cross-sectional view of the arrangement is shown.

- P and Q are two Josephson Junctions arranged in parallel.

When current I flows through the point C, it divides into I_1 and I_2

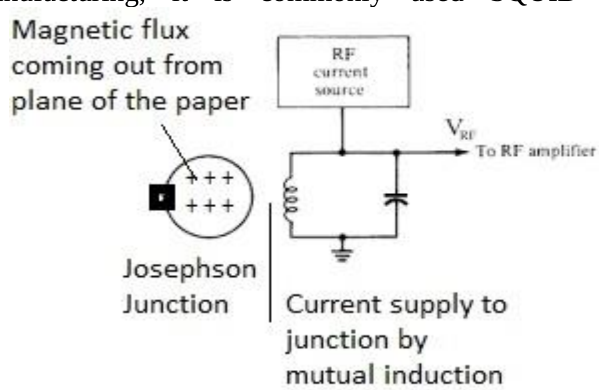


Cross Sectional View

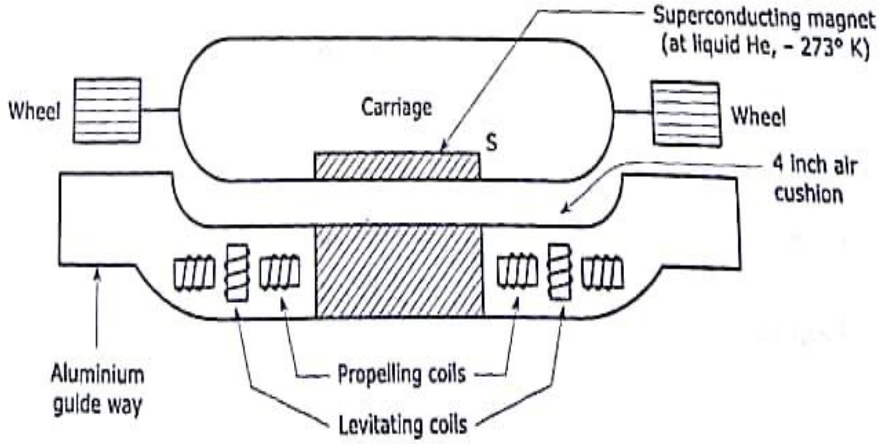
- Hence the wave function due to these super currents (cooper pairs) experiencea phase shift at P and Q
- In the absence of applied magnetic field, the phase difference between the wave functions is zero. If the magnetic field is applied perpendicular to the current loop, then phase difference between the wave functions will not be zero. This is identifiedby the sum of the currents I_1' and I_2'
- The magnitude of phase difference is proportional to applied magnetic field. Hence, even if there is a weak magnetic field in the region will be detected.

RF SQUID

- It works on the principle of AC Josephson effect - When dc voltage is applied across the Josephson junction, it leads to the development of oscillating current.
- It has single Josephson Junction.
- Magnetic field is applied perpendicular to theplane of the current loop.
- The flux is coupled into a loop containing a single Josephson Junction through an input coil and an RF source. Hence when the RF current changes, there is correspondingchange in the flux linked with the coil
- This variation is very sensitive and ismeasured.
- It is also used in the detection of low magnetic field.
- It is less sensitive compared to DC Squid.
- Due to its low-cost manufacturing, it is commonly used SQUID in many applications.



Maglev Vehicles:



- Magnetically Levitated Vehicle are called Maglev vehicles.
- They are made use of in transportation by being set afloat above a guideway. The utility of such a levitation is that, in the absence of contact between the moving and stationary systems, the friction is eliminated. With such an arrangement, great speeds could be achieved with very low energy consumption. The phenomenon on which the magnetic levitation is based is Meissner effect.
- The vehicle consists of superconducting magnets built into its base as shown in figure.
- There is an aluminium guide way over which the vehicle will be set afloat by magnetic levitation.
- The magnetic levitation is brought about by enormous repulsion between two highly powerful magnetic fields; one produced by the superconducting magnet inside the vehicle, and the other one by the electric currents in aluminium guideway.
- The currents in the guide way not only produce the necessary magnetic field to levitate the vehicle, but also help in propelling the vehicle forward.
- The vehicle is also provided with retractable wheels. The wheels serve almost the same purpose as those of an airplane. With the wheels, the vehicle runs on the guideway the way the airplane does during it take off. Once it is levitated in air, the wheels are retracted into the body. The height at which the vehicle is levitated above the guideway is about 10 to 15 cm. While stopping, the wheels are drawn out and the vehicle slowly settles on the guideway by running over a distance, as an airplane does while landing.