NAGARJUNA COLLEGE	**NAGARJUNACOLLEGEOFENGINEERINGANDTECHNOLOGY** **(Anautonomous institutionunderVTU)** DepartmentOfMathematics First Semester (23MATS11/23MATE11/23MATC11) Question Bank	Academic Year 2023-24

MODULE-1

DIFFERENTIAL CALCULUS

	Questions	CO	M	BL
1.	Derive angle between radius vector and the tangent to the polar curve r=f(θ).	CO1	7	L2
2.	Prove that with usual notations (i) $p = r\sin\varphi$ (ii) $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$.	CO1	6	L2
3.	Derive radius of curvature in Cartesian form.	CO1	6	L2
4.	Derive radius of curvature in parametric form.	CO1	7	L2
5.	Derive radius of curvature in polar form.	CO1	7	L2
6.	Derive radius of curvature in Pedal form.	CO1	6	L2
7.	Show that the curves $r^n\cos(n\theta)=a^n$ and $r^n\sin(n\theta)=b^n$ are orthogonal.	CO1	6	L2
8.	Show that the curves r=a(1+sinθ) and r=a(1−sinθ)intersect each other orthogonally.	CO1	6	L2
9.	Show that curves$r^n = a^n\cos(n\theta)$ and $r^n = b^n\sin(n\theta)$ cut each other orthogonally.	CO1	6	L2
10.	Find the angle of intersection between the following curves$r = a\log\theta$ and $r = \frac{a}{\log\theta}$.	CO1	7	L2
11.	Find the angle of intersection between the curves$r = \frac{a}{1+\cos\theta}$ and $r = \frac{b}{1-\cos\theta}$.	CO1	6	L2
12.	Find the angle of intersection between the curves$r = 4\sec^2(\theta/2)$ and $r = 9\,\mathrm{cosec}^2(\theta/2)$.	CO1	6	L2
13.	Find the angle of intersection between the curves$r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$.	CO1	6	L2
14.	Find the pedal equation of the curve $r = a(1 + \cos\theta)$.	CO1	6	L2
15.	Find the pedal equation of the curve$\frac{l}{r} = 1 + e\cos\theta$.	CO1	7	L2
16.	Find the pedal equation of the curve $r^m=a^m(\cos m\theta+\sin m\theta)$.	CO1	6	L2
17.	Find radius of curvature at (a,2a) of the parabola $y^2=4ax$.	CO1	6	L2
18.	Find radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ for the curve $x^3 + y^3 = 3axy$.	CO1	7	L2
19.	Show that radius of curvature at the point (a, 0) for the curve $y^2 = \frac{a^2(a-x)}{x}$ is $\frac{a}{2}$.	CO1	7	L2

20.	Show that radius of curvature at any point on the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ is $4a.\cos\left(\frac{\theta}{2}\right)$.	CO1	7	L2
21.	Find ρ at $\theta = \frac{\pi}{4}$ for the curve $x^{2/3} + y^{2/3} = a^{2/3}$. where $x = a\cos^3\theta$, $y = a\sin^3\theta$.	CO1	7	L2
22.	Show that ρ at any point of the cardiod $r = a(1 - \cos\theta)$ varies as $\sqrt{r}$.	CO1	7	L2
23.	Show that the radius of curvature of a curve $r^n = a^n \cos n\theta$ varies inversely as r^{n-1}.	CO1	7	L2
24.	Show that ρ at (r, θ)on the curve $r^2 = a^2 \sec 2\theta$ is proportional to r^3.	CO1	7	L2
25.	For the cardioid $r = a(1 + \cos\theta)$, show that $\frac{\rho^2}{r}$ is constant.	CO1	7	L2

MODULE-2

SERIES EXPANSION AND MULTI VARIABLE CALCULUS

	QUESTIONS	CO	M	BL
1	Expand $\mathbf{\log_e x}$ in powers of $(\boldsymbol{x - 1})$ and hence evaluate $(\mathbf{1.1})$ correct to four decimal places using Taylor's series Expansion.	CO1	6	L2
2	Find the Taylor's series expansion of $f(\boldsymbol{x}) = \mathbf{\log}\ \boldsymbol{cosx\ at\ x} = \frac{\pi}{3}$ up to fourth degree term.	CO1	6	L2
3	Expand $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$ up to the term containing $\left(x - \frac{\pi}{2}\right)^4$. Hence find the value of $\sin 91^0$ correct to 4 decimal places.	CO1	6	L2
4	Expand $\tan^{-1} x$ in powers of $(x - 1)$ up to the term containing fourth degree.	CO1	7	L2
5	Using Maclaurin's series, expand $y = \sqrt{(1 + \sin 2x)}$ in powers of x up to the term containing x^4.	CO1	6	L2
6	Find the Maclaurin's series of $\log(1 + x)$. Hence deduce that $\log\sqrt{\frac{1+x}{1-x}} = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots\ldots$	CO1	7	L2
7	Prove that $e^{\sin x} = 1 + x + \frac{x^2}{2!} - 3\frac{x^4}{4!} - \cdots\ldots\ldots$	CO1	7	L3
8	Prove that $\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \text{--------------------}$	CO1	7	L3
9	Evaluate (i) $\lim_{x\to1} \frac{x^x - x}{x - 1 - \log x}$ (ii) $\lim_{x\to0}\left(\frac{a^x + b^x + c^x}{3}\right)^{\frac{1}{x}}$	CO1	7	L2

10	Evaluate (i) $\lim_{x\to 0} \frac{\tan x - x}{x^2 \tan x}$ (ii) $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$	CO1	7	L2
11	Evaluate $\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}} - e}{x}$	CO1	6	L2
12	Find the values of a, b, c such that $\lim_{x\to 0} \frac{x(a+b\cos x)-c\sin x}{x^5} = 1$	CO1	7	L3
13	If $(x+y)z = x^2+y^2$, prove that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$	CO1	6	L2
14	If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$	CO1	7	L3
15	If $z = e^{ax+by} f(ax - by)$, prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$	CO1	6	L2
16	If $u = \log(x^3+y^3+z^3-3xyz)$, prove that (i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$ and (ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$	CO1	7	L2
17	If $f = x^2+y^2+z^2$ and $x = e^{2t}$, $y = e^{2t}\cos 3t$, $z = e^{2t}\sin 3t$, find $\frac{df}{dt}$	CO1	7	L2
18	If $z = f(x,y)$ and $x = e^u + e^{-v}$, $y = e^{-u} - e^v$ prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$	CO1	7	L2
19	If $u = f(x-y,\ y-z,\ z-x)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$	CO1	6	L2
20	If $u = x^2 - 2y^2$, $v = 2x^2 - y^2$, where $x = r\cos\theta$ and $y = r\sin\theta$, then show that $\frac{\partial(u,v)}{\partial(x,y)} = 6r^2\sin 2\theta$.	CO1	7	L2
21	If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, -1, 0).	CO1	7	L2
22	If $x = r\sin\theta\cos\emptyset$, $y = r\sin\theta\sin\emptyset$, $z = r\cos\theta$, then show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)} = r^2\sin\theta$.	CO1	7	L3
23	Discuss the maxima and minima of the function $f(x,y) = x^3y^2(1-x-y)$.	CO1	7	L3
24	Examine the function $f(x,y) = \sin x + \sin y + \sin(x+y)$ for extreme values.	CO1	7	L3
25	A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimension of the box requiring least material for its construction.	CO1	7	L3

MODULE 3

Ordinary Differential Equations Of First Order

	Questions	CO	Marks	BL
1.	Solve $\frac{dy}{dx} - \frac{2y}{x} = x + x^2$.	CO2	07	L2
2.	Solve $\frac{dy}{dx} + y\cot x = 4x\,\mathrm{cosec}\,x$ if $y = 0$ when $x = \frac{\pi}{2}$.	CO2	07	L2
3.	Solve $x\frac{dy}{dx} + y = x^3y^6$.	CO2	07	L2
4.	Solve $xy\,(1 + x\,y^2)\frac{dy}{dx} = 1$.	CO2	07	L2
5.	Solve $\left[y\left(1 + \frac{1}{x}\right) + \cos y\right] dx + [x + \log x - x\sin y]\,dy = 0$.	CO2	07	L2
6.	Solve $(2xy + y - \tan y)\,dx + (x^2 - x\tan^2 y + \sec^2 y)\,dy = 0$.	CO2	07	L2
7.	Solve $y(2x - y + 1)\,dx + x(3x - 4y + 3)\,dy = 0$.	CO2	07	L2
8.	Solve $(y\log y)\,dx + (x - \log y)\,dy = 0$.	CO2	07	L2
9.	Solve $(xy^3 + y)\,dx + 2\,(x^2y^2 + x + y^4)\,dy = 0$.	CO2	07	L2
10.	Show that the family of parabolas $y^2 = 4a(x + a)$ is self orthogonal.	CO2	07	L3
11.	Show that the family of conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$, Where λ is a parameter is self orthogonal.	CO2	07	L3
12.	Find the orthogonal trajectories of the family of curves $r^n = a^n \sin n\theta$, where a is the parameter.	CO2	07	L3
13.	Find the orthogonal trajectories of the family of cardiods $r = a(1 - \cos\theta)$, where a is the parameter.	CO2	07	L3
14.	If the temperature of the air 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C.	CO2	07	L3
15.	A body originally at 80°C cools down to 60°C in 20 minutes. If the air temperature is 40°C , what will be the temperature of the body after 40 minutes?	CO2	07	L3
16.	Water at temperature 10°C takes 5 minutes to warm up to 20°C in a room temperature at 40°C.Find the temperature after 20 minutes.	CO2	07	L3

17.	A bottle of mineral water at a room temperature of 72°F is kept in a refrigerator where the temperature is 44°F . After half an hour, water cooled to 61°F. How long will it take to cool to 50°F?	CO2	07	L3
18.	Obtain general and singular solution of $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$.	CO2	06	L2
19.	Obtain general and singular solution of $x\,y\left(\frac{dy}{dx}\right)^2 - (x^2 + y^2)\frac{dy}{dx} + xy = 0$.	CO2	06	L2
20.	Obtain general and singular solution of $p^2 + 2py \cot x = y^2$.	CO2	06	L2
21.	Obtain general and singular solution of $p(p + y) = x(x + y)$.	CO2	06	L2
22.	Find the general solution of the equation $(px - y)\,(py + x) = 2p$ by reducing in to Clairaut's form by taking the substitutions $X = x^2, Y = y^2$.	CO2	06	L2
23.	Find the general solution of the equation $(p - 1)\,e^{3x} + p^3 e^{2y} = 0$ by reducing in to Clairaut's form by taking the substitutions $u = e^x, v = e^y$.	CO2	06	L2
24.	Find the general solution of the equation $(px - y)\,(py + x) = a^2 p$ by reducing in to Clairaut's form by taking the substitutions $X = x^2, Y = y^2$.	CO2	06	L2
25.	Find the general solution of the equation $(\sin y \cos^2 x) = (\cos^2 y\, p^2) + (\sin x\ \cos x\ \cos y\, p)$ by reducing in to Clairaut's form by taking the substitutions $u = \sin y, v = \sin x$.	CO2	06	L2

Module 4

INTEGRAL CALCULUS

Questions	CO's	M	BL
1. Evaluate $\int_0^1 \int_{x^2}^{x} (x^2 + 3y + 2)\, dy\, dx$	CO3	6	L2
2. Evaluate $\int_0^1 \int_0^{\sqrt{x}} xye^{x^2}\, dy\, dx$	CO3	6	L2
3. Evaluate $\int_0^{\pi/2} \int_0^{a \sin\theta} r^3 \sin^2\theta\ \, dr\, d\theta$	CO3	6	L2
4. Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)\, dy\, dx$	CO3	6	L2
5. Evaluate $\int_{-1}^{1} \int_0^{z} \int_{x-z}^{x+z} (x + y + z)\, dx\, dy\, dz$	CO3	6	L2
6. Evaluate $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2)\, dz\, dy\, dx$	CO3	6	L2

7. Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}dx\,dy\,dz$	CO3	7	L2
8. Evaluate $\int_0^{\frac{\pi}{2}}\int_0^{a\sin\theta}\int_0^{\frac{a^2-r^2}{a}} r\;dz\,dr\,d\theta$	CO3	6	L2
9. Evaluate $\iint xy(x+y)dy\,dx$ taken over the area between $y = x^2$ and $y = x$	CO3	7	L2
10. Evaluate $\iint_A xy\;dx\,dy$ Where A is the domain bounded by ordinate $x = 2a$ and the curve $x^2 = 4ay$	CO3	7	L2
11.Evaluate $\iint_R x^2y\;dx\,dy$ where R is the region bounded by the lines, $y = x$, $x + y = 2$ and $y = 0$	CO3	7	L2
12.Evaluate $\iint y\,dx\,dy$ over the region by the first quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1.$	CO3	7	L2
13.Evaluate $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$ by changing the order of integration.	CO3	7	L3
14.Evaluate $\int_0^{\infty}\int_x^{\infty}\frac{e^{-y}}{y}dy\,dx$ by changing the order of integration	CO3	7	L3
15. Evaluate $\int_0^{a}\int_y^{a}\frac{x\,dx\,dy}{x^2+y^2}$ by changing the order of integration.	CO3	7	L3
16. Evaluate $\int_0^{\infty}\int_0^{x} xe^{-x^2/y}dy\,dx$ by changing the order of integration.	CO3	7	L3
17.Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dx\,dy$ by changing into polar coordinates. Hence show that $\int_0^{\infty} e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$.	CO3	7	L3
18. Evaluate $\int_0^{2}\int_0^{\sqrt{2x-x^2}}\frac{x}{x^2+y^2}\;dy\,dx$ by changing into polar coordinates.	CO3	7	L3
19. Evaluate $\int_0^{a}\int_0^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2}\;dx\,dy$ by changing to polar coordinates.	CO3	7	L3
20.Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$	CO3	7	L2
21. Prove that $\Gamma(1/2) = \sqrt{\pi}$ and show that $\int_0^{\infty} e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$.	CO3	7	L2
22.Evaluate i) $\int_0^{\pi/2}\sqrt{\cot\theta}\;d\theta$ ii) $\int_0^{\pi/2}\sqrt{\tan\theta}\;d\theta$	CO3	6	L2
23. Prove that $\int_0^{1}\frac{dx}{\sqrt{1+x^4}} = \frac{1}{4\sqrt{2}}\beta\left(\frac{1}{4},\frac{1}{2}\right)$.	CO3	7	L2
24.Prove that $\int_0^{1}\frac{dx}{\sqrt{1-x^4}} = \frac{\pi}{4}\frac{\Gamma(1/4)}{\Gamma(3/4)}$.	CO3	7	L2
25. Prove that $\int_0^{\frac{\pi}{2}}\sqrt{\sin\theta}\;d\theta * \int_0^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{\sin\theta}} = \pi.$	CO3	7	L2

MODULE-5

LINEAR ALGEBRA

	Questions	CO	M	BL
1.	Find the rank of the matrix $\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$ by reducing it to echelon form.	CO4	6	L2
2.	Find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ by reducing it to echelon form.	CO4	6	L2
3.	Find the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ by reducing it to echelon form.	CO4	6	L2
4.	Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & 8 \end{bmatrix}$ by reducing into echelon form.	CO4	6	L2
5.	Test the consistency and Solve $x + y + z = 6$, $x - y + 2z = 5$, $3x + y + z = 8$.	CO4	6	L2
6.	Test the consistency and Solve $x + 2y + 3z = 14$, $4x + 5y + 7z = 35$, $3x + 3y + 4z = 21$.	CO4	6	L2
7.	Test the consistency and Solve $x - 4y + 7z = 14$, $3x + 8y - 2z = 13$, $7x - 8y + 26z = 5$	CO4	6	L2
8.	Investigate the values of λ and μ so that the equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ may have a) Unique solution b) Infinite solution c) No solution	CO4	7	L2
9.	Solve the system of linear equations $2x_1 + 4x_2 + x_3 = 3$, $3x_1 + 2x_2 - 2x_3 = -2$, $x_1 - x_2 + x_3 = 6$ using Gauss elimination method.	CO4	6	L2
10.	Find the Solution of the system of linear equations using Gauss elimination method $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$.	CO4	6	L2
11.	Find the Solution of the system of linear equations using Gauss elimination method: $x + 4y - z = -5$, $x + y - 6z = -12$, $3x - y - z = 4$.	CO4	6	L2
12.	Apply Gauss-Jordan method to solve the equation $2x_1 + x_2 + 3x_3 = 1$, $4x_1 + 4x_2 + 7x_3 = 1$, $2x_1 + 5x_2 + 9x_3 = 3$.	CO4	6	L2
13.	Apply Gauss-Jordan method to solve the system of equation $2x + 5y + 7z = 52$, $2x + y - z = 0$, $x + y + z = 9$.	CO4	7	L2

14.	Apply Gauss-Jordan method to solve the system of equation $10x - 7y + 3z + 5u = 6\,, -6x + 8y - z - 4u = 5\,, 3x + y + 4z + 11u = 2\,,$ $5x - 9y - 2z + 4u = 7$	CO4	**7**	L2
15.	Find the Solution of system of linear equation using Gauss–Seidel method $20x + y - 2z = 17,\ \ 3x + 20y - z = -18,\ \ 2x - 3y + 20z = 25.$	CO4	**7**	L3
16.	Solve the equations $9x - y + 2z = 9,\ \ x + 10y - 2z = 15,\ \ -2x + 2y + 13z = 17$ using Gauss–Seidel method by taking (1, 1, 1) as initial approximate solution .	CO4	**7**	L3
17.	Solve the equations $\begin{bmatrix} 5 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -6 \end{bmatrix}$ using Gauss–Seidel method.	CO4	**7**	L3
18.	Find the eigen value and the eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$	CO4	**7**	L2
19.	Find the eigen values and the eigen vectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$	CO4	**7**	L2
20.	Find the eigen values and the eigen vectors of the matrix $\begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$	CO4	**7**	L2
21.	Find the Eigen values and eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$	CO4	**7**	L2
22.	Find the Largest Eigen value and the corresponding Eigen vector of the matrix A = $\begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$ by the power method.Perform five iteration. Take $[1\,,0\,,0]^T$ as initial approximation.	CO4	**7**	L3
23.	Find the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by using power method.	CO4	**7**	L3
24.	Find the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ using the power methodby taking the initial approximation to the eigen vector as $[1, 0.8\,, -0.8]^T$. Perform five iterations.	CO4	**7**	L3
25.	Determine the largest (dominant) eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ using the power method. Taking the initial Eigenvector as $[1,0,0]'$	CO4	**7**	L3