NAGARJUNA COLLEGE OF ENGINEERING AND TECHNOLOGY
Department of Mathematics

Even Semester 2023-24
Internal Assessment Test – I

Course Name: Advanced Calculus and Numerical Methods	**Course Code:** **23MATS21/23MATE21/23MATC21**	**Semester: II**
Date: 24/04/2024	**Time: 9:30 - 10:30am /** **11:00am - 12:00pm**	**Max. Marks: 25**

[Note: Answer any ONE full question from each PART]

<table>
<tr><th>Sl. No</th><th></th><th>PART-A
QUESTIONS</th><th>COs</th><th>RBT Levels</th><th>Marks</th></tr>
<tr><td>1.</td><td>a)</td><td>Find the angle between the tangent planes to the surfaces $xlogz = y^2 - 1$ and $x^2y = 2 - z$ at the point (1,1,1).</td><td>CO1</td><td>L2</td><td>05</td></tr>
<tr><td></td><td>b)</td><td>Prove that $\vec{A} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational and find a scalar function f(x,y,z) such that $\vec{A} = \nabla f$.</td><td>CO1</td><td>L3</td><td>05</td></tr>
<tr><td></td><td></td><td>(OR)</td><td></td><td></td><td></td></tr>
<tr><td>2.</td><td>a)</td><td>Find the directional derivative of $\emptyset = x^2yz + 4xz^2$ at the point (1,-2,1) in the direction of the vector $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$.</td><td>CO1</td><td>L2</td><td>05</td></tr>
<tr><td></td><td>b)</td><td>Evaluate $\int[(xy + y^2)dx + x^2dy]$.where c is the boundary by $y = x$ and $y = x^2$, using Green's Theorem.</td><td>CO1</td><td>L3</td><td>05</td></tr>
<tr><td></td><td></td><td>PART-B</td><td></td><td></td><td></td></tr>
<tr><td>3.</td><td>a)</td><td>Find the real root of the equation $cosx = 3x - 1$ correct to three decimal places using the method of False position.</td><td>CO2</td><td>L2</td><td>05</td></tr>
<tr><td></td><td>b)</td><td>The area A of a circle of diameter d is given for the following values:
<table>
<tr><td>d</td><td>80</td><td>85</td><td>90</td><td>95</td><td>100</td></tr>
<tr><td>A</td><td>5026</td><td>5674</td><td>6362</td><td>7088</td><td>7854</td></tr>
</table>
Find the area of the circle of the diameter 105 by using suitable interpolation formula.</td><td>CO2</td><td>L3</td><td>05</td></tr>
<tr><td></td><td></td><td>(OR)</td><td></td><td></td><td></td></tr>
<tr><td>4.</td><td>a)</td><td>Applying Lagranges's formula find y when x=3 given the data
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>5</td></tr>
<tr><td>f(x)</td><td>2</td><td>3</td><td>12</td><td>147</td></tr>
</table></td><td>CO2</td><td>L2</td><td>05</td></tr>
<tr><td></td><td>b)</td><td>Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Trapezoidal rule by taking 6 sub intervals.</td><td>CO2</td><td>L3</td><td>05</td></tr>
<tr><td></td><td></td><td>PART-C</td><td></td><td></td><td></td></tr>
<tr><td>5.</td><td>a)</td><td>Evaluate divergence of $2x^2z\,\hat{\imath} - xy^2z\hat{\jmath} + 3yz^2\hat{k}$.</td><td>CO1</td><td>L2</td><td>03</td></tr>
<tr><td></td><td>b)</td><td>State Green's theorem in the plane.</td><td>CO1</td><td>L1</td><td>02</td></tr>
<tr><td></td><td></td><td>(OR)</td><td></td><td></td><td></td></tr>
<tr><td>6.</td><td>a)</td><td>Using Newton-Raphson method ,find the real root of the equation $x^3 + x^2 + 3x + 4 = 0$ perform two iteration which is near $x = -1$.</td><td>CO2</td><td>L2</td><td>03</td></tr>
<tr><td></td><td>b)</td><td>State Simpson's one third rule.</td><td>CO2</td><td>L1</td><td>02</td></tr>
</table>

NAGARJUNA COLLEGE OF ENGINEERING AND TECHNOLOGY

Department of Mathematics

Even Semester 2023-24

Internal Assessment Test – II

Course Name: Advanced Calculus and Numerical Methods	**Course Code:** 23MATS21/23MATE21/23MATC21	**Semester: II**
Date: 11/06/2024	**Time: 9:30-10:30am / 11:00-12:00pm**	**Max.Marks: 25**

[**Note:** Answer any **ONE** full question from each **PART**]

Sl. No		PART-A	COs	RBT Levels	Marks
1.	a.	Employ Taylor's method to obtain approximate value of y at x = 0.2 correct to four decimal places for the differential equation $\frac{dy}{dx} = 2y + 3e^x,\ y(0) = 0$.	CO2	L3	5M
	b.	Use Modified Euler's Method to solve $\frac{dy}{dx} = \log(x + y), y(0) = 2$ at $x = 0.2$ correct to four decimal places with $h = 0.2$. (Use modified formula twice).	CO2	L3	5M
		(OR)			
2.	a.	Using Runge-Kutta method of order four, find $y(0.2)$ correct to four decimal places for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$. Take $h = 0.2$.	CO2	L3	5M
	b.	Given $y' = x - y^2$, using the data $y(0) = 0,\ y(0.2) = 0.02,\ y(0.4) = 0.0795,\ y(0.6) = 0.1762$. Compute y at x = 0.8 correct to four decimal places by applying Milne's method. (Use Corrector formula twice)	CO2	L3	5M
		PART-B			
3.	a.	Solve $(D^2 + 2D + 1)\,y = x^2 + 2x$. Imm	CO3	L2	5M
	b.	Using the method of variation of parameter, solve $\frac{d^2y}{dx^2} + a^2y = \text{cosec}\,ax$.	CO3	L3	5M
		(OR)			
4.	a.	Solve $\frac{d^3y}{dx^3} + y = 0$. Hom	CO3	L2	5M
	b.	Solve $x^2y'' - xy' + y = \log x$. Cauchy	CO3	L3	5M
		PART-C			
5.	a.	Form the Partial Differential Equation by eliminating arbitrary constants from $z = a\log(x^2 + y^2) + b$.	CO4	L2	5M
		(OR)			
6.	a.	Form the Partial Differential Equation by eliminating the arbitrary function from $f(x^2 + y^2, z - xy) = 0$.	CO4	L2	5M

NAGARJUNA

COLLEGE OF ENGINEERING AND TECHNOLOGY

NAGARJUNA COLLEGE OF ENGINEERING AND TECHNOLOGY
(An autonomous institution under VTU)
Department Of Mathematics
Second Semester
CCE1-Questions

Academic Year
2023-24

Note: Each Question carries 10 Marks

1\. a Prove that $\vec{A} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational and find a scalar function $f(x, y, z)$ such that $\vec{A} = \nabla f$.

b If $\vec{F} = 3xy\hat{\imath} - y^2\hat{\jmath}$, then evaluate $\int_c \vec{F}.d\vec{r}$, where C is the curve in the xy - plane given by $y = 2x^2$ from (0, 0) to (1, 2).

2\. a Show that $\vec{F} = (y + z)i + (z + x)j + (x + y)k$ is irrotational. Also find a scalar function $\emptyset$ such that $\vec{F} = \nabla\emptyset$.

b If $\vec{A} = (3x^2 + 6y)\,\hat{\imath} - 14yz\,\hat{\jmath} + 20xz^2\,\hat{k}$, evaluate $\int_c \vec{A}.d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path $x = t$, $y = t^2$, $z = t^3$.

3\. a Evaluate curl $\vec{F}$ where $\vec{F} = grad(x^3y + y^3z + z^3x - x^2y^2z^2)$.

b Find the total work done by the force $\vec{F} = 3xy\hat{\imath} - y\hat{\jmath} + 2zx\hat{k}$ in moving a particle around the circle $x^2 + y^2 = 4$.

4\. a Find the values of a and b such that the surfaces $5x^2 - 2yz - 9z + 9 = 0$ may cut the surface $ax^2 + by^3 = 4$ orthogonally at the point $(1, -1, 2)$.

b Use Green's Theorem to evaluate $\int_c xydx + x^2y^3dy$, where C is the triangle with vertices (0,0), (1,0), (1,2) with positive orientation.

5\. a Evaluate curl $\vec{F}$ where $\vec{F} = grad[x^3 + y^3 + z^3 - 3xyz]$.

b Using Green's theorem to evaluate $\int_c [(y - \sin x)dx + \cos x\,dy]$ where C is the plane triangle enclosed by the lines $y = 0$, $x = \pi/2$ and $y = 2x/\pi$.

6\. a Find the directional derivative of $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ where Q is the point $(5, 0, 4)$. Also calculate the magnitude of the maximum directional derivative.

b Apply Green's theorem to evaluate $\int_c [(2x^2 - y^2)dx + (x^2 + y^2)\,dy]$ where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$.

7\. a If $\vec{A} = (x + y + az)\hat{\imath} + (bx + 2y - z)\hat{\jmath} + (x + cy + 2z)\hat{k}$ find a, b and c such that curl $\vec{A} = 0$ (i.e., $\vec{A}$ is irrotational).

b Use Green's Theorem to evaluate $\int_c [x^2y\,dx + x^2dy]$ where C is the boundary described counter clockwise of triangle with vertices (0,0) , (1,0) , (1,1).

8\. a Find the directional derivative of $\emptyset = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the of the normal to the surface $x\log z - y^2 = -4$ at $(-1,2,1)$

b Use Stoke's theorem to evaluate $\int_c \vec{F}.d\vec{r}$ where $\vec{F} = y^2\hat{\imath} + x^2\hat{\jmath} - (x + z)\hat{k}$ and C is the boundary of the triangle with vertices at (0, 0, 0), (1, 0, 0) and (1, 1, 0).

9\. a Find the values of the a and b such that $\vec{F} = (axy + z^3)\,\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3bxz^2 - y)\,\hat{k}$ is irrotational.

b Apply Stoke's theorem to evaluate $\int_c [y\,dx + zdy + x\,dz]$ where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$.

10\. a Find the angle between the tangent planes to the surfaces $x\log z = y^2 - 1$ and $x^2y = 2 - z$ at the point $(1, 1, 1)$

b Evaluate $\int_c [(xy + y^2)dx + x^2dy]$, where C is bounded by $y = x$ and $y = x^2$, using Green's Theorem.

NAGARJUNA

Nagarjuna College of Engineering & Technology

(Autonomous Institute Affiliated to VTU)

Second Semester BE Degree SE Examination, July- 2024

Advanced Calculus and Numerical Methods

Time: 3Hrs. Max. Marks: 100

Note: Answer any one full question from each module.

COs M BL

Module - 1

1a Find the unit vector normal to the surface $xy^3z^2 = 4$ at the point $(-1,-1,\ 2)$. CO1 6 L2

b Evaluate $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$ where $\vec{F} = grad[x^3 + y^3 + z^3 - 3xyz]$. CO1 7 L2

c Using Green's theorem to evaluate $\int_c [(y - sinx)dx + cosx\, dy]$ where C is the plane triangle enclosed by the lines $y = 0$, $x = \pi/2$ and $y = 2x/\pi$. CO1 7 L3

OR

2a Find the directional derivative of $\emptyset = x^2yz + 4xz^2$ at the point $(1,-2,\ 1)$ in the direction of the vector $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$. CO1 6 L2

b Find the values of a and b such that the surfaces $5x^2 - 2yz - 9z + 9 = 0$ may cut the surface $ax^2 + by^3 = 4$ orthogonally at the point $(1,-1,\ 2)$. CO1 7 L2

c Find the total work done by the force $\vec{F} = 3xy\hat{\imath} - y\hat{\jmath} + 2zx\hat{k}$ in moving a particle around the circle $x^2 + y^2 = 4$. CO1 7 L3

Module - 2

3a Using Newton – Raphson method find the real root of $xsinx + cosx = 0$ correct to three decimal places which is near $x = \pi$. CO2 6 L3

b The area A of a circle of diameter d is given for the following values:

d	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area of the circle of the diameter 105 by using suitable interpolation formula. CO2 7 L3

c Evaluate $\int_0^1 \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's three eighth rule. Hence deduce the value of log 2. CO2 7 L3

OR

4a Use Regula Falsi method to find the real root of $xe^x = cosx$ correct to four decimal places which lies in the interval (0, 1). Carry out three iterations. CO2 6 L3

b Given the values

x	5	7	11	13	17
$f(x)$	150	392	1452	2366	5202

Evaluate $f(9)$ using Lagrange's Interpolation formula. CO2 7 L3

c Use Simpson's one third rule to find $\int_0^{0.6} e^{-x^2}$ dx by taking 6 sub intervals. CO2 7 L3

Module - 3

5a Employ Taylor's series method to obtain approximate value of y at $x = 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x,\ y(0) = 0$. CO2 6 L3

b Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with y(0)=1 at x = 0.2 correct to four decimal places by taking h = 0.2. CO2 7 L3

c Given $y' = x - y^2$, using the data $y(0) = 0,\ y(0.2) = 0.02,\ y(0.4) = 0.0795,$ $y(0.6) = 0.1762,$ compute y at x = 0.8 correct to four decimal places by applying Milne's method. Apply the corrector formula twice. CO2 7 L3

OR

6a	Find the value of y at $x = 0.1$ to five places of decimals by Taylor's series method from $\frac{dy}{dx} = x^2y - 1,\ y(0) = 1.$	CO2	6	L3
b	Apply Runge-Kutta method of fourth order to solve $\frac{dy}{dx} = \frac{2xy+e^x}{x^2+xe^x}$ at x = 1.2 given $y_0 = 0$ when $x_0 = 1$ correct to four decimal places by taking h = 0.2.	CO2	7	L3
c	Using Modified Euler's Method find $y(0.1)$ correct to four decimal places given that $\frac{dy}{dx} = x - y^2,\ y(0) = 1$. Taking h = 0.1, use modified formula twice.	CO2	7	L3

Module - 4

7a	Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)\,y = 0$.	CO3	6	L2
b	Find the general solution of $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$.	CO3	7	L2
c	Obtain the solution of $x^2y'' + xy' + 9y = 3x^2 + \sin(3\log x)$.	CO3	7	L3

OR

8a	Obtain y by solving $(D^2 + 2D + 1)y = x^2 + 2x$.	CO3	6	L2
b	Find the general solution of $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$.	CO3	7	L2
c	Using the method of variation of parameter solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$.	CO3	7	L3

Module - 5

9a	Form a partial differential equation by eliminating the arbitrary constants a, b, c from the relation $z = ax + by + cxy$.	CO4	6	L2
b	Solve $\frac{\partial^2 z}{\partial x^2} = xy$ subject to the condition that $\frac{\partial z}{\partial x} = \log(1+y)$ when x = 1 and z = 0 when x = 0.	CO4	7	L2
c	Find all various possible solutions of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables.	CO4	7	L3

OR

10a	Form a partial differential equation by eliminating the arbitrary function from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$.	CO4	6	L2
b	Solve the equation $\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial y} - 6z = 0$ given that $z = x$ and $\frac{\partial z}{\partial y} = 0$, when $y = 0$.	CO4	7	L2
c	Solve by the method of separation of variables $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ where $u(0, y) = 2e^{5y}$.	CO4	7	L3