NAGARJUNA COLLEGE OF ENGINEERING AND TECHNOLOGY

Department of Mathematics

Odd Semester 2023-24

Internal Assessment Test – II

Course Name: Calculus and Linear Algebra	Course Code: 23MATS11/23MATE11/23MATC11	Semester: I
Date: 29.12.2023	Time: 09.30AM to 10.30AM	Max. Marks: 25

[Note: Answer any one full question from each part]

Q. No.		QUESTIONS	COs	RBT Levels	Marks
		PART-A			
1.	a	Find the solution of $\frac{dy}{dx} + y\cot x = 4x \operatorname{cosec} x$	CO2	L2	04M
	b	Show that the family of parabolas $y^2 = 4a(x+a)$ is self orthogonal	CO2	L3	06M
		OR			
2.	a	Solve $\left[y\left(1+\frac{1}{x}\right)+\cos y\right] dx + [x + \log x - x\sin y]\, dy = 0.$	CO2	L2	04M
	b	A body originally at 80°C cools down to 60°C in 20 minutes. If the air temperature is 40°C, what will be the temperature of the body after 40 minutes?	CO2	L3	06M
		PART-B			
3.	a	Find the rank of the matrix $\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$ by reducing it to echelon form	CO4	L2	04M
	b	Solve the equations $9x - y + 2z = 9$, $x + 10y - 2z = 15$, $-2x + 2y + 13z = 17$ using Gauss–Seidel method by taking $(1, 1, 1)$ as initial approximate solution.	CO4	L3	06M
		OR			
4.	a	Obtain the Solution of the system of linear equations using Gauss elimination method $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$.	CO4	L2	04M
	b	Find the Largest Eigen value and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$ by the power method. Perform five iterations. Take $[1, 0, 0]^T$ as initial approximation.	CO4	L3	06M
		PART-C			
5.		Prove that $\Gamma(1/2) = \sqrt{\pi}$.	CO3	L1	05M
		OR			
6.		Show that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta * \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} = \pi.$	CO3	L1	05M

Nagarjuna College of Engineering & Technology

(Autonomous Institute Affiliated to VTU)

First Semester BE Degree SE Examination, January 2024

CALCULUS AND LINEAR ALGEBRA

Time: 3Hrs. Max. Marks: 100

Note: Answer any one full question from each module

Q. No.	Question	COs	M	BL
	Module - 1			
1a	Show that the curves $r=a(1+\sin\theta)$ and $r=a(1-\sin\theta)$ intersects each other orthogonally.	CO1	6	L2
b	Derive the expression for radius of curvature in polar form.	CO1	7	L2
c	Find the pedal equation of the curve $\frac{l}{r} = 1 + e\cos\theta$.	CO1	7	L2
	OR			
2a	Find the pedal equation of the curve $r = a(1+\cos\theta)$.	CO1	6	L2
b	Derive the formula for angle between radius vector and the tangent to the polar curve $r = f(\theta)$.	CO1	7	L2
c	Show that the radius of curvature at the point $(a, 0)$ for the curve $y^2 = \frac{a^2(a-x)}{x}$ is $\frac{a}{2}$.	CO1	7	L2
	Module - 2			
3a	If $z = e^{ax+by} f(ax-by)$, prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$.	CO1	6	L2
b	If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, -1, 0).	CO1	7	L2
c	Prove that $e^{\sin x} = 1 + x + \frac{x^2}{2!} - 3\frac{x^4}{4!} - \cdots\cdots\cdots$	CO1	7	L3
	OR			
4a	If $(x+y)z = x^2 + y^2$, prove that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$.	CO1	6	L2
b	Evaluate (i) $\lim_{x\to 1} \frac{x^x - x}{x - 1 - \log x}$ (ii) $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$.	CO1	7	L2
c	Discuss the maxima and minima of the function $f(x,y) = x^3y^2(1-x-y)$.	CO1	7	L3
	Module - 3			
5a	Solve $\left[y\left(1+\frac{1}{x}\right) + \cos y\right]dx + [x + \log x - x\sin y]\,dy = 0$.	CO2	6	L2
b	Obtain the solution of $p^2 + 2py\cot x = y^2$.	CO2	7	L2
c	Find the orthogonal trajectories of the family of curves $r^n = a^n \sin n\theta$, where a is the parameter.	CO2	7	L3
	OR			
6a	Solve $\frac{dy}{dx} + y\cot x = 4x\,\mathrm{cosec}\,x$ if $y = 0$ when $x = \frac{\pi}{2}$.	CO2	6	L2
b	Find the general solution of the equation $(p-1)e^{3x} + p^3e^{2y} = 0$ by reducing in to Clairaut's form by taking the substitutions $u = e^x$, $v = e^y$.	CO2	7	L2
c	If the temperature of the air 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C.	CO2	7	L3
	Module - 4			
7a	Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta * \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$.	CO3	6	L2
b	Evaluate $\int_{-1}^{1}\int_0^z\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$	CO3	7	L2
c	Find $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$ by changing the order of integration.	CO3	7	L3
	OR			

8a	FInd $\int_0^{\pi/2} \int_0^{a \sin\theta} r^3 \sin^2\theta \, dr \, d\theta$.	CO3	6	L2
b	Evaluate $\iint xy(x+y)dy\,dx$ taken over the area between $y = x^2$ and $y = x$.	CO3	7	L2
c	Obtain $\int_0^a \int_0^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2} \, dx\,dy$ by changing to polar coordinates.	CO3	7	L3

Module - 5

9a	Find the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ by reducing it into echelon form.	CO4	6	L2
b	Investigate the values of λ and μ so that the equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ may have (i) Unique solution (ii) Infinite solution (iii) No solution.	CO4	7	L2
c	Find the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ using the power method by taking the initial approximation to the eigen vector as $[1, 0.8, -0.8]^T$. Perform five iterations.	CO4	7	L3

OR

10a	Find the Solution of the system of linear equations using Gauss elimination method: $x + 4y - z = -5$, $x + y - 6z = -12$, $3x - y - z = 4$.	CO4	6	L2
b	*Find the eigen values and the eigen vectors of the matrix* $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$	CO4	7	L2
c	Solve the equations $9x - y + 2z = 9$, $x + 10y - 2z = 15$, $-2x + 2y + 13z = 17$ using Gauss–Seidel method by taking (1, 1, 1) as initial approximate solution.	CO4	7	L3