MODULE-02

(RL)

Regular language: The language which is accepted by finite automata.

Regular Expression: The declarative way of representing the RL.

1. Find RL for following Regular Expr.

i) RE : a^{+} $\bigcup_{i=1}^{\infty}(L)^i$

$\bigcup_{i=1}^{\infty}(a)^i$

$RL = \{a^1 \cup a^2 \cup a^3 \cup \ldots\}$

$RL = \{a, aa, aaa \ldots\}$

ii) RE : b^{*} $\bigcup_{i=0}^{\infty}(L)^i \Rightarrow \bigcup_{i=0}^{\infty}(b)^i$

$RL = \{(b)^0 \cup (b)^1 \cup (b)^2 \cup (b^3) \cup \ldots\}$

$RL = \{\epsilon, b, bb, bbb\} \Rightarrow \{b, bb, bbb\}$

iii) RE = $a^{+}b^{*}$

$RL = \{(a)^1(b)^0, (a)^2(b)^1, (a)^3(b)^2 \ldots\}$

$= \{a\epsilon, aab, aaabb \ldots\} \Rightarrow \{a, aab, aaabb\}$

iv) RE : $a^{+}b^{+}a^{*}b^{*}$

$RL = \{(a)^1(b)^1(a)^0(b)^0, (a)^2(b)^2(a)^1(b)^1, (a)^3(b)^3(a)^2(b)^2$

$= \{ab\epsilon\epsilon, aabbab, aaabbbaabb \ldots\}$

$= \{ab, aabbab, aaabbbaabb \ldots\}$

	RL	RE
1.	$\{\phi\}$	ϕ
2.	$\{\epsilon\}$	ϵ
3.	$\{a, b\}$	$(a+b)$
4	$\{ab\}$	ab
5.	$\{a, b, aa, ba, ab, bb\}$	$(a+b)^{+}$
6.	$\{\epsilon, ab, aa, ab, ba, bb\}$	$(a+b)^{*}$

RE	RL
1. $a^+a^*b^+ab$	$\{abab, aaabbab\ldots\}$
2. $a^+b^*a^+b^*$	$\{aa, aabaab, \ldots\}$
3. $abb^*b^+a^+$	$\{abba, abbbbaa, \ldots\}$
4. $ab^+b^*a^+$	$\{aba, abbbaa, \ldots\}$

Properties of R.E:

If r, s, t are RE then the following properties are true

Here, $r = s$ means $L(r) = L(s)$

1. $r + s = s + r$
2. $(r+s)+t = r+(s+t)$
3. $(rs)t = r(st)$
4. $r(s+t) = rs + rt$.
5. $(r+s)t = rt + st$
6. $\phi^* = \epsilon$
7. $(r^*)^* = r^*$
8. $(\epsilon + r)^* = r^*$
9. $(r^* s^*)^* = (r+s)^*$

Algebraic Laws for RE: (or) Identity rules for RE:

2 RE p, q are equivalent if p, q represent the same set of strings.

1. $\phi + r = r$
2. $\phi r + r\phi = \phi$
3. $\epsilon r = r \epsilon = r$
4. $\epsilon^* = \epsilon$
5. $r + r = r$
6. $r^* r^* = r^*$
7. $rr^* = r^* r$
8. $(r^*)^* = r^*$
9. $\epsilon + rr^* = \epsilon + r^* r = r^*$
10. $(pq)^* p = p(qp)^*$
11. $(p+q)^* = (p^* q^*)^*$
12. $(p+q)r = pr + qr$

12 *) $\epsilon + r = r$

13) $r(p+q) = rp + rq$

NOTE :

Especially all identities rules formulas will be used for minimizing (simplifying) the RE

Q) $\underbrace{(1+00^*1)}_{a} + \underbrace{(1+00^*1)}_{a}\underbrace{(1+10^*1)^*}_{b}\underbrace{(1+10^*1)}_{c} = 0^*1(1+10^*1)^*$

$\rightarrow$ $\underbrace{(1+00^*1)}_{a}\left(\epsilon + \underbrace{(1+10^*1)^*}_{b}\underbrace{(1+10^*1)}_{c}\right)$

$(1+00^*1)(1+10^*1)^*$

$1(\epsilon+00^*)(1+10^*1)^*$

$\boxed{0^*1(1+10^*1)^*}$

(6) 9

$\boxed{\epsilon + r^*r = r^*}$

$r = 1+10^*1$

$1(\epsilon+00^*)$

$\epsilon + rr^* = r^*$

Q) $\epsilon + \underline{a^*(abb)^*}\,(\underline{a^*(abb^*)})^* = (a+abb)^*$

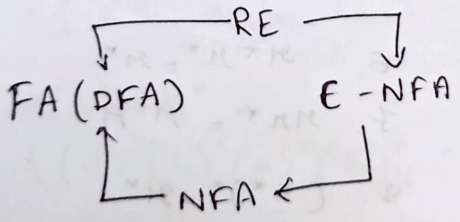
$\rightarrow$ $\epsilon + rr^* = r^*$

$(a^*(abb)^*)^*$ $\Rightarrow$ $(r^*s^*)^* = (r+s)^*$

$(a+abb)^*$ (or) $(p+r)^* = (p^*q^*)^*$

Construction of finite automata (DFA / NFA)

The relation b/w FA and RE is shown below

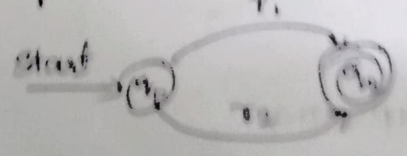


while converting RE to FA we need to follow 3 Expression

1. Union Expression 2. Concatenation 3. kleen closure

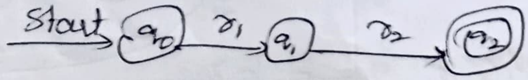
↳ let R_1, R_2 be a 2 RE's then the following notation shows the union of 2 RE's

$r_1 + r_2$



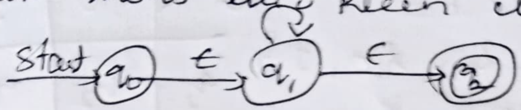
2) Concatation let r_1, r_2 be 2 RE's then following notation shows the concatination of RE.

$r_1 r_2$



3) Kleen closure : let r, be a RE then following notation shows the r kleen closure

r^*



$r^* = \{\epsilon, r, rr, rrr \ldots\}$

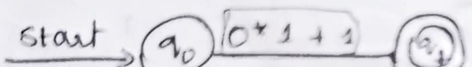
$\epsilon . \epsilon = \epsilon$

$\epsilon . r . \epsilon = r$

$\epsilon . rr . \epsilon = rr$

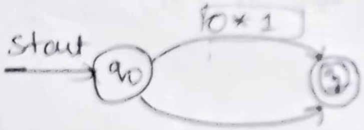
Q) construct FA for following RE. = 0^*1+1

→ RE 0^*1+1



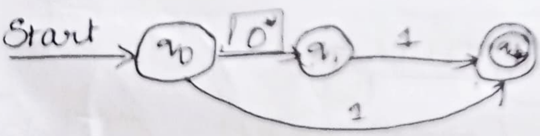
let $r_1 = 0^*1$, $r_2 = 1$ } $r_1 + r_2$

we need to apply union notation

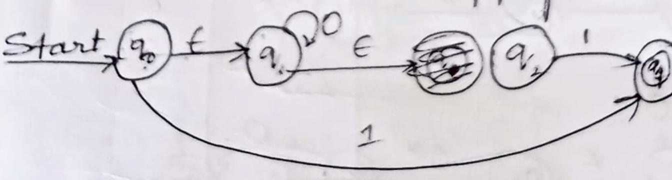


0^*1

let $r_1 = 0^*$; $r_2 = 1$ then apply concatenation notation



Let $r = 0$; $r^* = 0^*$ then apply kleen closure notation



	0	1	ϵ
q_0	ϕ	q_f	q_1
q_1	q_2	ϕ	q_2
q_2	ϕ	q_f	ϕ
q_f	ϕ	ϕ	ϕ

ϵ - closure : ϵ-closure$(q_0) = q_0 \cup q_1 \cup q_2$ ⇒ $q_0 q_1 q_2$

ϵ-closure$(q_1) = q_1 \cup q_2$ ⇒ $q_1 q_2$

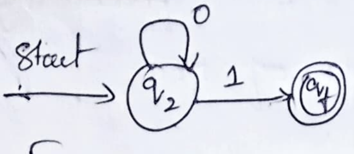
ϵ-closure

Conversion : $\delta(Q, \Sigma) \to \epsilon$-closure$(\delta(\epsilon$-closure$(Q)\Sigma)$

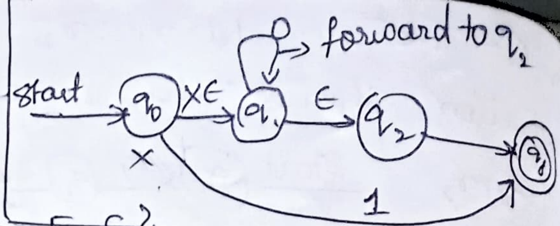
$\delta(q_0\ 1) \to \epsilon$-closure$(\delta(\epsilon$-closure$(q_0)\ 1)$

ϵ-closure$(\delta(q_0 q_1 q_2)\ 1)$

$\delta(q_0\ 1) = \epsilon$-closure$(q_f)$ ⇒



(DFA)



Formal def. $M = \{Q, \Sigma, q_0, F, \delta\}$

$Q = \{q_2, q_f\}$; $\Sigma = \{0, 1\}$

$q_0 = \{q_2\}$; $F = \{q_f\}$

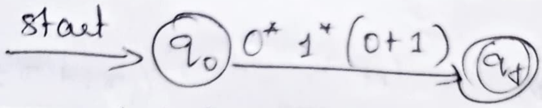
$\delta = Q \times \Sigma = \{q_2\, q_f\} \times \{0, 1\}$

$\delta(q_2, 0) = q_2$; $\delta(q_2, 1) = q_f$

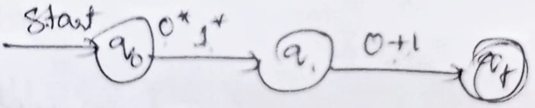
$Q \backslash \Sigma$	0	1
q_2	q_2	q_f
q_f^*	dead	dead state

2) $0^* 1^* (0+1)$ construct FA (DFA) for following RE.

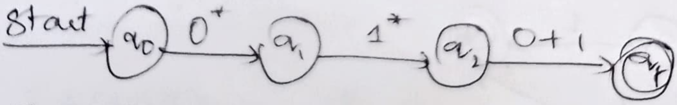
RE = $0^* 1^* (0+1)$



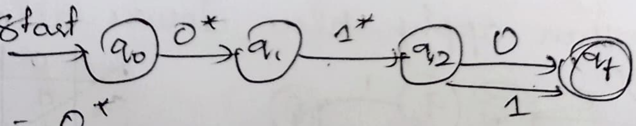
$r_1 r_2 \Rightarrow 0^* 1^* (0+1)$ $r_1 = 0^* 1^*$ $r_2 = (0+1)$



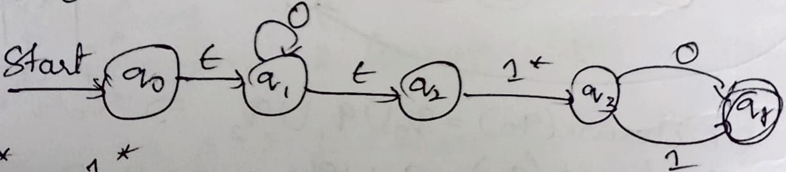
$r_1 r_2$: $r_1 = 0^*$ $r_2 = 1^*$



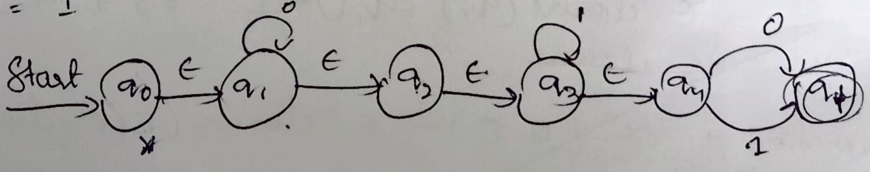
$r_1 + r_2$ = $r_1 = 0$ $r_2 = 1$

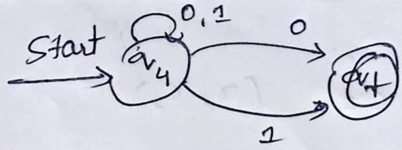


$r_1^* = 0^*$



$r^* = 1^*$





(NFA)

Formal def

$M = \{Q, \Sigma, q_0, F, \delta\}$

$Q = \{q_4\ q_f\}$ $\Sigma = \{0, 1\}$

$q_0 = \{q_4\}$, $q_F = \{q_f\}$

Transition: $\delta = \{Q \times \Sigma\}$

$\delta = \{q_4\ q_f\} \times \{0, 1\}$

$\delta(q_4, 0) = q_4, q_f$

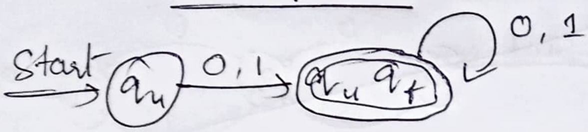
$\delta(q_4, 1) = q_4, q_f$

$Q \backslash \Sigma$	0	1
$\rightarrow q_4$	q_4, q_f	q_4, q_f
q_f^*	ϕ	ϕ

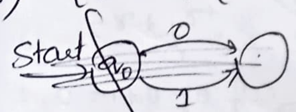
Equivalent STT of DFA

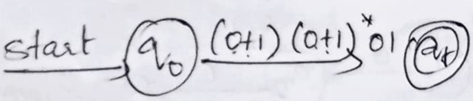
$Q \backslash \Sigma$	0	1
$\rightarrow q_4$	$q_4 q_f$	$q_4 q_f$
$q_4 q_f^*$	$q_4 q_f$	$q_4 q_f$

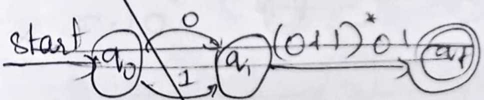
STD of DFA



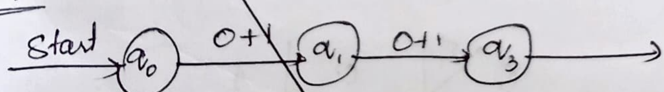
3) $(0+1)(0+1)^* 01$ $\quad \cancel{r_1 + r_2 = r_1 = 0}\ \ r_2 = 1$





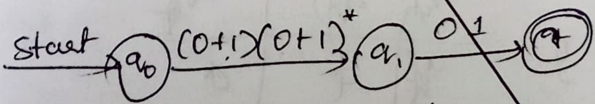


$r_1 r_2$ $\quad r_1 = 0+1 \quad r_2 = 0+1$



3) $RE = (0+1)(0+1)^* 01$

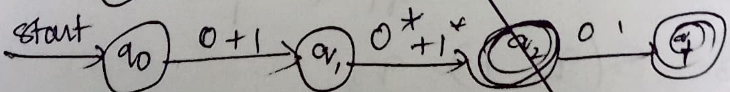
$r_1 = (0+1)(0+1)^* \quad r_2 = 01$



$r_1 r_2$

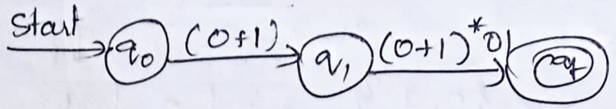
$r_1 = 0+1$

$r_2 = 0^* + 1^*$

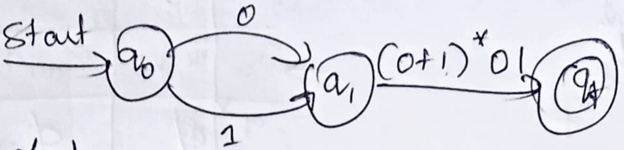


3) RE = (0+1) (0+1)* 01

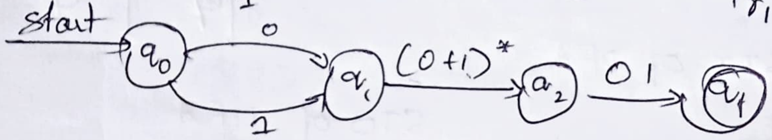
→ $r_1 = 0+1$ $r_2 = (0+1)^* 01$ $(r_1 r_2)$



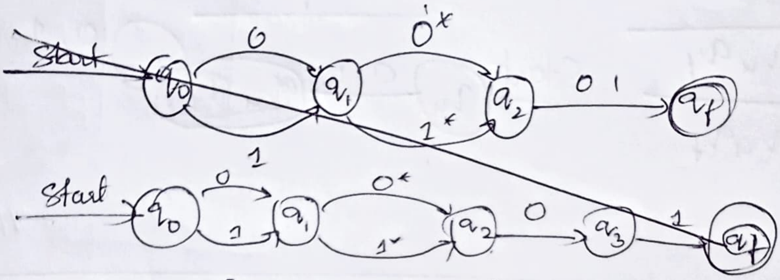
$r_1 + r_2$
0   1



$r_1 r_2$
$r_1 = (0+1)^*$ $r_2 = 01$

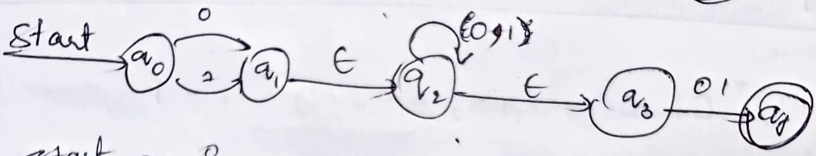


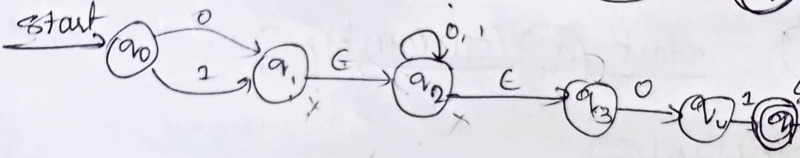
$r_1 + r_2$
~~$r_1 = 0^*$ $r_2 = 1^*$~~



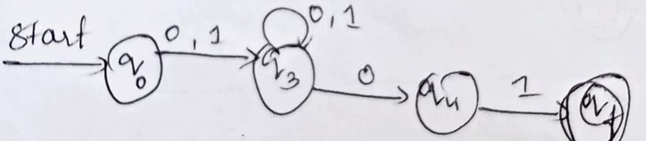
~~$r_1 r_2$~~
~~$r_1 = 0$ $r_2 = 1$~~

r^*
$(0+1)^*$





$r_1 + r_2$
0   1
self of 0+1 = 0,1
$01 = r_1 r_2$



(NFA)

Formal def :

$M = \{Q, \Sigma, q_0, F, \delta\}$
$Q = \{q_0, q_3, q_4, q_f\}$
$\Sigma = \{0, 1\}$
$q_0 = \{q_0\}$; $F = \{q_f\}$

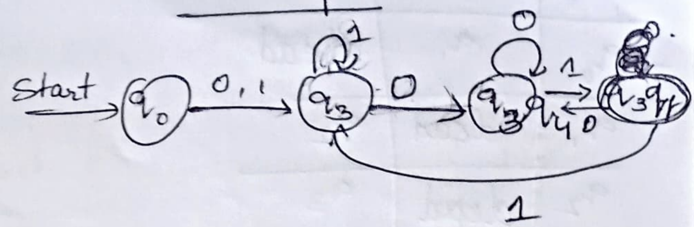
Q \ Σ	0	1
→ q_0	q_3	q_3
q_3	q_3, q_4	q_3
q_4	ϕ	q_f
q_f^*	ϕ	ϕ

$\delta(q_0, 1) = q_3$; $\delta(q_0, 1) = q_3$
$\delta(q_3, 0) = q_3, q_4$; $\delta(q_3, 1) = q_3$
$\delta(q_4, 0) = \phi$; $\delta(q_4, 1) = q_f$

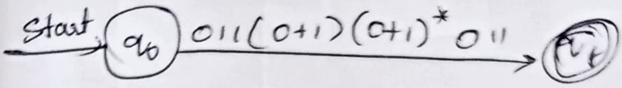
Equivalent STT of DFA

$Q \backslash \Sigma$	0	1
$\rightarrow q_0$	q_3	q_3
q_3	$q_3 q_4$	q_3
$q_3 q_4$	$q_3 q_4$	q_3, q_f
$q_3 q_f$	$q_3 q_4$	q_3

STD of DFA



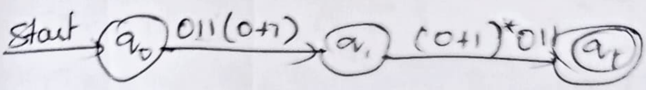
4) $011(0+1)(0+1)^*011$



$r_1 r_2$

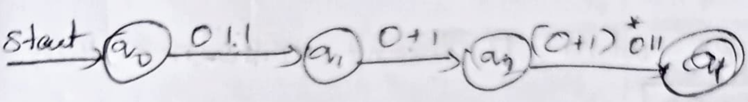
$r_1 = 011(0+1)$

$r_2 = (0+1)^* 011$



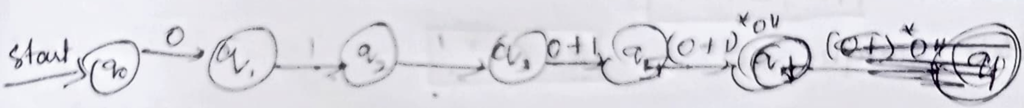
$r_1 . r_2$

$r_1 = 011 \quad r_2 = (0+1)$



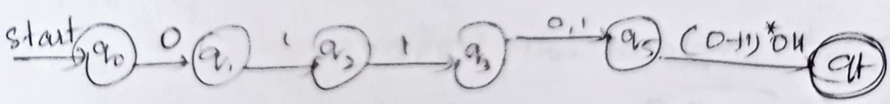
$r_1 r_2 r_3$

0 1 1



$r_1 + r_2$

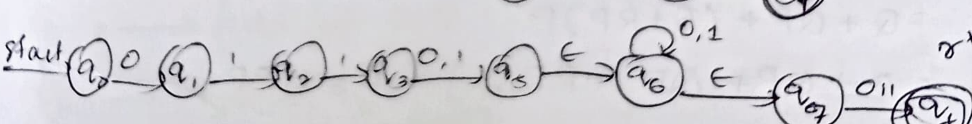
0 + 1

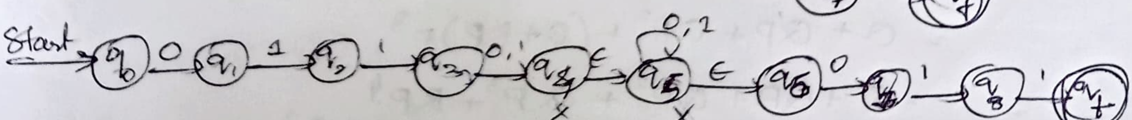


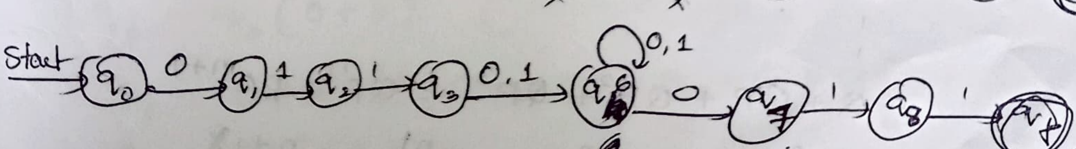
$r_1 . r_2$

$r_1 = (0+1)^* \quad r_2 = 011$

$r^* = (0+1)^*$







(NFA)

$M = \{Q, \Sigma, q_0, F, \delta\}$

$Q = \{q_0, q_1, q_2, q_3, q_6, q_7, q_8, q_f\}$

$\Sigma = \{0, 1\}$

$q_0 = \{q_0\}; \quad F = \{q_f\}$

$\delta(q_0, 0) = q_1, \quad \delta(q_0, 1) = \phi$

$Q \backslash \Sigma$	0	1
$\rightarrow q_0$	q_1	ϕ
q_1	ϕ	q_2
q_2	ϕ	q_3
q_3	q_6	q_6
q_6	q_6, q_7	q_6
q_7	ϕ	q_8
q_8	ϕ	q_f
q_f	ϕ	ϕ

$Q \backslash \Sigma$	0	1
$\rightarrow q_0$	q_1	~~dead~~ dead
q_1	~~dead~~ dead	q_2
q_2	~~dead~~ dead	q_3
q_3	q_6	q_6
q_6	$q_6 q_7$	q_6
$q_6 q_7$	$q_6 q_7$	$q_6 q_8$
$q_6 q_8$	$q_6 q_7$	$q_6 q_f$
$q_6 q_f^*$	$q_6 q_7$	q_6

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Arden's theorem

Let 'P', 'Q', 'R' be a RE where, P doesn't contain ϵ, then equation is $\boxed{R = Q + RP}$

For this, unique soln as $\boxed{R = QP^*}$

Proof: $R = Q + RP$

R can be replaced with $Q+RP$

$R = Q + (Q+RP)P$

$= Q + QP + RP^2$ $*\ R = Q + RP$

$= Q + QP + (Q+RP)P^2$

$= Q + QP + QP^2 + RP^3$ $*\ R = Q + RP$

$= Q + QP + QP^2 + (Q+RP)P^3$

$= Q + QP + QP^2 + QP^3 + RP^4$

⋮

$= Q + QP + QP^2 + QP^3 \ldots QP^n + RP^{n+1}$

$= Q(\epsilon + P + P^2 + \ldots P^n) + RP^{n+1}$

$= Q(\epsilon \cup P \cup P^2 \cup \ldots P^n) + RP^{n+1}$ $\boxed{+ \Rightarrow \cup}$

~~* kleen closure~~

* kleen closure $= QP^* + RP^{n+1}$ - discard

(recursively Extending)

$\{P, P^2, P^3, P^4 \ldots P^{n+1}\}$

$\boxed{R = QP^*}$

→ Construct RE reverse process:

Construct RE for following FA:

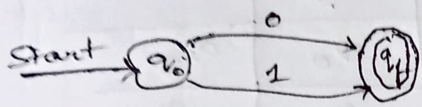
NOTE :- 1. According to the Arden's theorem we can write Arden's equation for each & every state in a given diagram.

2. For starting state of Arden's eq^n we must add 'ϵ'

3. We can apply Arden's theorem start only for final state of eq^n.

4. It's applicable only for NFA's & DFA's

①



(DFA)

→ Formal def of DFA

$M = \{Q, \Sigma, q_0, F, \delta\}$

$Q = \{q_0, q_f\}$

$\Sigma = \{0, 1\}$

$q_0 = \{q_0\}$; $F = \{q_f\}$

STT of DFA

Q \ Σ	0	1
→ q_0	q_f	q_f
q_f*	dead	dead

Transition Fun^n

$\delta = Q \times \Sigma$

$\{q_0, q_f\} \times \{0, 1\}$

$\delta(q_0, 0) = q_f$; $\delta(q_0, 1) = q_f$

Arden's eq^n

$q_0 \longrightarrow$ (add ϵ + ϕ (there's no incoming arrow mark to q_0) → ①

$q_f \longrightarrow q_0 0 + q_0 1$ (incoming arrow (input)) → ②

from eq^n ②

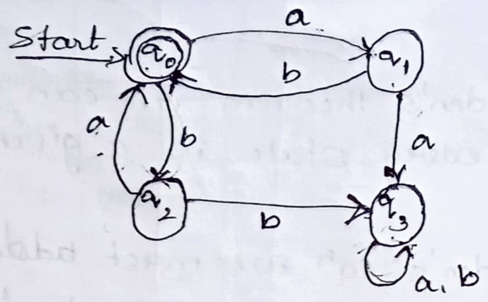
$q_f = q_0(0+1)$ (common q_0)

$q_f = \epsilon + \phi\,(0+1)$ → $q_0 = \underbrace{\epsilon + \phi}_{\epsilon}$ from eq^n ①

$q_f = \epsilon\,(0+1)$

$q_f = (0+1)$

(2) Construct RE for following FA :



(DFA)

→ **Formal def**

$M = \{Q, \Sigma, q_0, F, \delta\}$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

$q_0 = \{q_0\}$; $F = \{q_0\}$

~~From~~

$\delta = Q \times \Sigma$

$\delta(q_0, a) = q_1$; $\delta(q_0, b) = q_2$

$\delta(q_1, a) = q_3$; $\delta(q_1, b) = q_0$

$\delta(q_2, a) = q_0$; $\delta(q_2, b) = q_3$

$\delta(q_3, a) = q_3$; $\delta(q_3, b) = q_3$

STT

$Q \backslash \Sigma$	a	b
→ q_0*	q_1	q_2
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_3	q_3

Arden's eqn

$q_0 = \epsilon + q_1 b + q_2 a$ — (1)

$q_1 = q_0 a$ — (2)

$q_2 = q_0 b$ — (3)

$q_3 = q_1 a + q_2 b + q_3 a + q_3 b$ — (4)

~~$q_3 = q_3 a + q_3 b + q_1 a + q_2 b$~~ $q_3(a+b)$

$q_3 = q_1 a + q_2 b + q_3 a + q_3 b$ — (4) ✓

$q_0 = \epsilon + q_1 b + q_2 a$

~~$q_0 = \epsilon + (q_0 b) a + (q_0 a) b$~~

$= \epsilon + q_0 ab + q_0 ab$ $\epsilon + 2 q_0 ab$

$= \epsilon + q_0 ab (\epsilon + \epsilon)$

$q_0 = \epsilon + q_0 (ab + ba)$

$R = Q + R P$ *

$q_0 = \epsilon + q_1 b + q_2 a$

$q_0 = \epsilon + q_0 ab + q_0 ba$

$q_0 = \epsilon + \underline{q_0} (ab + ba)$

$* \; R = \underset{\uparrow \epsilon}{Q} + \underset{\uparrow q_0}{RP}$

$R = q_0$

$Q = \epsilon$

$P = (ab+ba)$

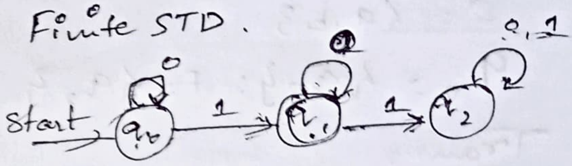
$R = \underline{QP^*}$

$q_0 = \epsilon . (ab+ba)^*$

$q_0 = (ab+ba)^*$

Q) Construct RE for following Finite STD.

$Q \backslash \Sigma$	0	1
$\rightarrow q_0$	q_0	q_1
q_1^*	q_1	q_2
q_2^*	q_2	q_2



STD

$\rightarrow$ Formal def

$M = \{Q, \Sigma, q_0, F, \delta\}$

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1\}$

$q_0 = \{q_0\}; \; F = \{q_1\}$

Tran : $\delta = Q \times \Sigma$

$\delta(q_0, 0) = q_0 ; \; \delta(q_0, 1) = q_1$

$\delta(q_1, 0) = q_1 ; \; \delta(q_1, 1) = q_2$

$\delta(q_2, 0) = q_2 ; \; \delta(q_2, 1) = q_2$

Arden's eqn

$q_0 = \epsilon + q_0 0 \rightarrow ①$

$q_1 = q_1 0 + q_0 1 \rightarrow ②$

$q_2 = q_1 1 + q_2 0 + q_2 1 \rightarrow ③$

$R = Q + RP \Rightarrow R = QP^*$

$q_0 = \epsilon + q_0 0$

$q_0 = \epsilon (0)^*$

$q_0 (0)^*$

$q_1 = q_0 1 + q_1 0$

$q_1 = (0)^* 1 + q_1 0$

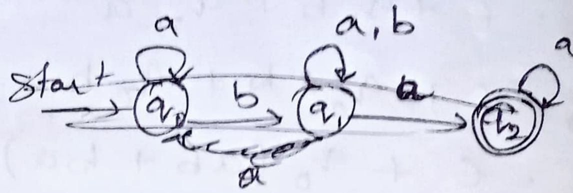
$R = Q + RP \Rightarrow QP^*$

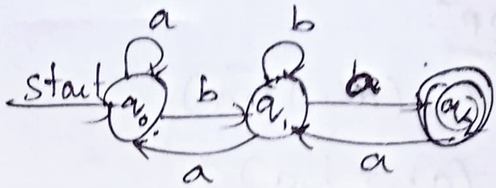
shd be same to apply arden's theorem

$\boxed{q_1 = (0)^* 1 (0)^*}$

Q)

$Q \setminus \Sigma$	a	b
$\rightarrow q_0$	q_0	q_1
q_1	$q_0 q_2$	q_2
q_2^*	q_2	ϕ





$\rightarrow$ $M = \langle Q, \Sigma, q_0, F, \delta \rangle$

$Q = \{q_0, q, q_2\}$

$\Sigma = \{a, b\}$

$q_0 = \{q_0\}$: $F = \{q_2\}$

Tran

$\delta = Q \times \Sigma$

$\delta(q_0, a) = q_0$: $\delta(q_0, b) = q_1$

$\delta(q, a) = q_1 q_2$: $\delta(q_1, b) = q_1$

$\delta(q_2, a) = q_2$: $\delta(q_2, b) = \phi$

Ardm's q?

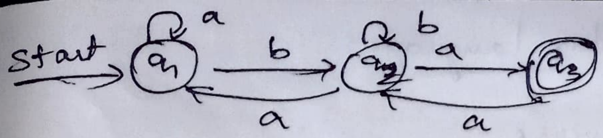
$q_0 = \epsilon + q_0 a + q_1 a$

$q_1 = q_0 b + q_1 b + q_2 a$ $\quad R = Q + RP$

$q_2 = q_2 a$ $\quad R = QP^*$

$q_0 = \epsilon + q_0 a + q_1 a$

$q_0 = \epsilon + a(q_0 + q_1)$



$Q \backslash \Sigma$	a	b
$\rightarrow q_1$	q_1	q_2
q_2	q_3	q_2
q_3^*	q_2	ϕ

$Q = \{q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

$q_0 = \{q_1\}$

$F = \{q_3\}$

$\delta(q_1, a) = q_1 \quad \delta(q_1, b) = q_2$

$\delta(q_2, a) = q_3 \quad \delta(q_2, b) = q_2$

$\delta(q_3, a) = q_2$

Arden's eqn

$q_1 = \epsilon + q_1 a + q_2 a \rightarrow (1)$

$q_2 = q_1 b + q_2 b + q_3 a \rightarrow (2)$

$q_3 = q_2 a \rightarrow (3)$

$(2) \Rightarrow q_2 = q_1 b + q_2 b + q_3 a$

$= q_1 b + q_2 b + q_2 a a$

$q_2 = q_1 b + q_2 (b + aa) \Rightarrow Q + RP$

$R = Q + RP \quad \Rightarrow QP^*$

$$\boxed{q_2 = q_1 b (b + aa)^*}$$

~~$q_3 = q_2 a$~~

~~$= q_1 b$~~

$(1) \Rightarrow q_1 = \epsilon + q_1 a + q_2 a$

$= \epsilon + q_1 a + q_1 b (b + aa)^* a$

$\epsilon + q_1 \{a + b(b + aa)^* a\}$

$\boxed{q_1 = \epsilon \{a + b(b + aa)^* a\}}$

$q_1 = (a + b(b + aa)^* a)^*$

$(3) \Rightarrow q_3 = q_2 a$

$= q_1 b (b + aa)^* a$

$q_3 = (a + b(b + aa)^* a)^* b (b + aa)^* a$

→ closure properties of Regular language:

1. Union
2. Concatenation
3. Intersection
4. Positive closure
5. kleen closure
6. Reverse
7. Compliment
8. Difference
9. Subsitution.
10. Homomorphism
11. Inverse Homo

① **Union**: The clause of closed under union. If L_1 & L_2 are 2 languages, then their union is denoted by $L_1 \cup L_2$ or L_1, L_2 or $L_1 + L_2$.

② **Concatenation**: The clause of closed under Conca. If L_1 and L_2 are 2 lang, then their Conca is denoted by $L_1 L_2$.

③ **Kleen closure**: the clause... If L be a language then it's kleen closure is denoted by L^*.

④ **Positive closure**: The clause.. denoted by L^+.

⑤ **Reverse**: L^R.

⑥ **Compliment**: L' or $\bar{L} = L^* - L$

⑦ **Intersection**: $L_1 \cap L_2$

⑧ **Difference**: $L_1 - L_2$

⑨ **Subsitution**: (Replacement)

⑩ **Homomorphism**: The clause of closed under Hom. Let L be a lang. Then it's denoted by $h(L)$.

⑪ **Inverse Hom**: $h^{-1}(L)$.

Applications of RE :

(5m)

1. RE used in Lexical Analyzer.
2. used in text editor.

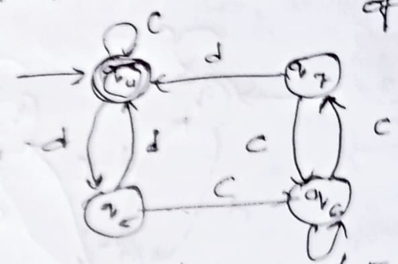
Security Application
Natural Lang processing
Text mining and data Extraction
Search and replace operation.

→ Pumming Lemma:

→ Equivalence of 2 FA's (F, F) (NF, NF) → equivalence of 2DFA's

FA1

FA2



⟹ Formal def of FA1

$M = \{Q, \Sigma, q_0, F, \delta\}$

$Q = \{q_1, q_2, q_3\}$

$\Sigma : \{c, d\}$

$q_0 : \{q_1\}$

$F : \{q_1\}$

Formal def of FA2:

$Q = \{q_4, q_5, q_6, q_7\}$

$\Sigma = \{c, d\}$

$q_0 = \{q_4\}$

$F = \{q_4\}$

STT of FA1

Q \ Σ	c	d
→q_1^*	q_1	q_2
q_2	q_3	q_1
q_3	q_2	q_3

STT of FA2.

Q \ Σ	c	d
→q_4^*	q_4	q_5
q_5	q_6	q_4
q_6	q_7	q_6
q_7	q_6	q_4

Transition fn FA1

$\delta = Q \times \Sigma$

$\delta(q_1, c) = q_1$; $\delta(q_1, d) = q_2$

$\delta(q_2, c) = q_3$; $\delta(q_2, d) = q_1$

$\delta(q_3, c) = q_2$; $\delta(q_3, d) = q_3$

T.F of FA2:

$\delta(q_4, c) = q_4$; $\delta(q_4, d) = q_5$

$\delta(q_5, c) = q_6$; $\delta(q_5, d) = q_4$

$\delta(q_6, c) = q_7$; $\delta(q_6, d) = q_6$

$\delta(q_7, c) = q_6$; $\delta(q_7, d) = q_4$

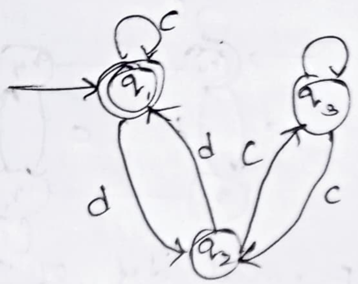
Equivalenat Table

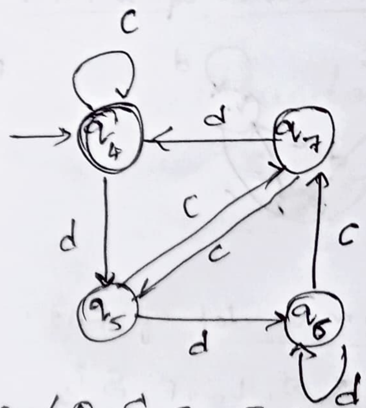
Initial state of 2 FA's

Q \ Σ	c	d
(q_1, q_4)	q_1 q_4 F F	q_2 q_5 NF NF
q_2 q_5	q_3 q_6 NF NF	q_1 q_4 F F
q_3 q_6	q_2 q_7 NF NF	q_3 q_6 NF NF
q_2 q_7	q_3 q_6 NF NF	q_1 q_4 F F

∴ the above 2 FA are equivalent

2) DFA 1





⟹ $M = \langle Q, \Sigma, q_0, F, \delta \rangle$

$Q = \{q_1, q_2, q_3\}$

$\Sigma = \{c, d\}$

$q_0 = \{q_1\}$; $F = \{q_1\}$

$M = \langle Q, \Sigma, q_0, F, \delta \rangle$

$Q = \{q_4, q_5, q_6, q_7\}$

$\Sigma = \{c, d\}$

$q_0 = \{q_4\}$; $F = \{q_4\}$

Q \ Σ	c	d
→ q_1*	q_1	q_2
q_2	q_3	q_1
q_3	q_2	q_3

Q \ Σ	c	d
→ q_4*	q_4	q_5
q_5	q_7	q_6
q_6	q_7	q_6
q_7	q_5	q_4

$\delta(q_1, c) = q_1$; $\delta(q_1, d) = q_2$

$\delta(q_2, c) = q_3$; $\delta(q_2, d) = q_1$

$\delta(q_3, c) = q_2$; $\delta(q_3, d) = q_3$

$\delta(q_4, c) = q_4$; $\delta(q_4, d) = q_5$

$\delta(q_5, c) = q_7$; $\delta(q_5, d) = q_6$

$\delta(q_6, c) = q_7$; $\delta(q_6, d) = q_6$

$\delta(q_7, c) = q_5$; $\delta(q_7, d) = q_4$

2) check whether the following language is RL or not

$L = \{ a^P \mid P \text{ is prime number} \}$ $P = 2, 3, 5, 7, 11 \ldots$

$\Rightarrow$ $p = a^2 = aa$; $a^5 = aaaaa$

$p = a^3 = aaa$; $a^7 = aaaaaaa$.

$L = \{ a^2, a^3, a^5 \ldots \}$

$= \{ aa, aaa, aaaaa \ldots \}$

any string. $w = aaa$

$w = xyz$ $y \notin \epsilon$

i) $x = a$; $y = a$; $z = a$ $\in L$

ii) $x = \epsilon$; $y = a$; $z = aa$

iii) $x = \epsilon$; $y = aa$; $z = a$

iv) $x = a$; $y = aa$; $z = \epsilon$

chose any case

iii) $x = \epsilon$; $y = aa$; $z = a$

$i \geq 0$ $xy^iz \Rightarrow \epsilon (aa)^0 (a) \Rightarrow a \notin L$

$i = 1$ $xy^1z \Rightarrow \epsilon (aa)^1 (a) \Rightarrow aaa \in L$

$i = 2$ $xy^2z \Rightarrow \epsilon (aa)^2 (a) \Rightarrow aaaaa \in L$

$i = 3$ $xy^3z \Rightarrow \epsilon (aa)^3 (a) \Rightarrow aaaaaaa \in L$

$i = 4$ $xy^4z \Rightarrow \epsilon (aa)^4 (a) \Rightarrow aaaaaaaaa \notin L$

$\therefore$ The given lang is not R.L.